Algebra: Expanding

Overview

This presentation will cover:

- Distributive Law
- Special expansions

Distributive Law

Consider the expression $5 \times (3 + 4)$.

This means that $3 + 4$ is multiplied by 5, which is equivalent to '5 lots of 3 plus 5 lots of 4'.

Let's check:

$$5 \times (3 + 4) = 5 \times 7 = 35$$

$$5 \times 3 + 5 \times 4 = 15 + 20 = 35$$

Writing this in mathematical notation,

$$5 \times (3 + 4) = 5 \times 3 + 5 \times 4\,.$$

Distributive Law — Algebra

Algebraically the Distributive Law is written:

$$a(b + c) = ab + ac$$

Try these expansion questions:

1. $2(x + 3)$
2. $-2(x - 4)$
3. $2x(x + 1)$

Solutions:

1.

$$
\begin{aligned}
2(x+3) &= 2\times x + 2\times 3 \\
&= 2x+6
\end{aligned}
$$

2.

$$
\begin{aligned}
-2(x-4) &= -2\times x - 2\times(-4) \\
&= -2x+8
\end{aligned}
$$

3.

$$
\begin{aligned}
2x(x+1) &= 2x\times x + 2x\times 1 \\
&= 2x^2+2x
\end{aligned}
$$

Expanding two brackets

We can now use this same skill to expand two brackets.

$$
\begin{aligned}
(x+a)(x+b) &= (x+a)x + (x+a)b \\
&= x\times x + a\times x + x\times b + a\times b \\
&= x^2 + ax + bx + ab \\
&= x^2 + (a+b)x + ab\,.
\end{aligned}
$$

Example, Expanding two brackets

$$
\begin{aligned}
(x+3)(x+4) &= (x+3)x + (x+3)4 \\
&= x^2 + 3x + 4x + 12 \\
&= x^2 + 7x + 12
\end{aligned}
$$

Exercise

1. $(x+3)(x-2)$
2. $(2x-1)(x+2)$
3. $(3a-2b)(4a+3b)$
4. $(x-2)(x^2-3x+4)$
5. $(2x-3y+z)(4x+2y-z)$

Solutions

1.

$$
\begin{aligned}
(x+3)(x-2) &= x(x-2)+3(x-2) \\
&= x^2-2x+3x-6 \\
&= x^2+x-6\,.
\end{aligned}
$$

2.

$$
\begin{aligned}
(2x-1)(x+2) &= 2x(x+2)-1(x+2) \\
&= 2x^2+4x-1x-1\times 2 \\
&= 2x^2+3x-2\,.
\end{aligned}
$$

Solutions (continued)

3.

$$
\begin{aligned}
(3a-2b)(4a+3b) &= 3a(4a+3b)-2b(4a+3b) \\
&= 12a^2+9ab-8ab-6b^2 \\
&= 12a^2+ab-6b^2\,.
\end{aligned}
$$

4.

$$
\begin{aligned}
(x-2)(x^2-3x+4) &= x(x^2-3x+4)-2(x^2-3x+4) \\
&= x^3-3x^2+4x-2x^2+6x-8 \\
&= x^3-5x^2+10x-8
\end{aligned}
$$

Solutions (continued)

5.

$$
\begin{aligned}
& (2x-3y+z)(4x+2y-z) \\
= {} & 2x(4x+2y-z)-3y(4x+2y-z)+z(4x+2y-z) \\
= {} & 8x^2+4xy-2xz-12xy-6y^2+3yz+4xz+2yz-z^2 \\
= {} & 8x^2-6y^2-z^2-8xy+2xz+5yz\,.
\end{aligned}
$$

Important expansions to remember

Some important examples worth remembering are,

1. $(x+a)^2 = x^2+2ax+a^2$
2. $(x-a)^2 = x^2-2ax+a^2$
3. $(x+a)(x-a) = x^2-a^2$

Verify these expansions yourself!

Expanding three brackets

Expand $(3y-1)(2y+2)(3-y)$.

$$\begin{aligned}
& (3y-1)(2y+2)(3-y) \\
= \ & (3y-1)[2y(3-y)+2(3-y)] \\
= \ & (3y-1)(6y-2y^2+6-2y) \\
= \ & (3y-1)(6+4y-2y^2) \\
= \ & 3y(6+4y-2y^2)-1(6+4y-2y^2) \\
= \ & 18y+12y^2-6y^3-6-4y+2y^2 \\
= \ & -6y^3+14y^2+14y-6\,.
\end{aligned}$$

Exercise

1. $(2x-1)(3x+2)(x-3)$
2. $(x^2+y^2)(x-y)(2x+y)$

Solutions

1.

$$\begin{aligned}
& (2x-1)(3x+2)(x-3) \\
= \ & (2x-1)[3x(x-3)+2(x-3)] \\
= \ & (2x-1)(3x^2-9x+2x-6) \\
= \ & (2x-1)(3x^2-7x-6) \\
= \ & 2x(3x^2-7x-6)-1(3x^2-7x-6) \\
= \ & 6x^3-14x^2-12x-3x^2+7x+6 \\
= \ & 6x^3-17x^2-5x+6\,.
\end{aligned}$$

Solutions (continued)

2.

$$\begin{aligned}
& (x^2+y^2)(x-y)(2x+y) \\
= \ & (x^2+y^2)(2x^2+xy-2xy-y^2) \\
= \ & (x^2+y^2)(2x^2-xy-y^2) \\
= \ & 2x^4-x^3y-x^2y^2+2x^2y^2-xy^3-y^4 \\
= \ & 2x^4-x^3y+x^2y^2-xy^3-y^4\,.
\end{aligned}$$

Summary

This presentation covered:

- Expanding brackets (1, 2, and 3)
- Special expansions to recognise