

## Introduction to Algebra

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### How to translate from words to symbols:



- ▶ read the problem twice;
- ▶ read the problem out aloud to yourself, if possible;
- ▶ ask yourself four questions:
  - ▶ What is the problem asking me?
  - ▶ What facts are given in the problem?
  - ▶ Are there any special conditions?
  - ▶ Is any information irrelevant?
- ▶ draw a diagram or picture, if appropriate;
- ▶ break the problem down into parts;
- ▶ define the variables;
- ▶ look for connections between variables;
- ▶ write the connection out in words, then write them out as an algebraic relationship.

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### Relationships as formulas: Exercise



Express the following relationships as formulas.

1. The grevillea is half the height of the palm tree.
2. The adult weighs three times as much as the child.
3. The perimeter of a square is four times the length of one side.
4. The house is 15 metres longer than it is wide.

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## Relationships as formulas: Answers



You may have different letters to those given here, but the formula should look the same and you should have defined the variables clearly and fully.

1. The grevillea was half the height of the palm tree.

$$G = \frac{1}{2}P,$$

where  $G$  represents the *height* of the grevillea, and  $P$  represents the *height* of the palm tree.

2. The adult weighed three times as much as the child.

$$\begin{aligned} A &= 3 \times C, \\ A &= 3C, \end{aligned}$$

where  $A$  represents the weight of the adult, and  $C$  represents the weight of the child.

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## Relationships as formulas: Answers (cont)



3. The perimeter of a square is four times the length of one side.

$$\begin{aligned} P &= 4 \times s, \\ P &= 4s, \end{aligned}$$

where  $P$  represents the perimeter, and  $s$  represents the length of one side.

4. The house was 15 metres longer than it was wide.

$$L = W + 15,$$

where  $L$  represents the length of the house, in metres, and  $W$  represents the width of the house, in metres.

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## Variables and expressions



Terms used in Algebra:

- ▶ A **variable** is a term that is used to indicate that that symbol may take various values.
  - ▶ Usually a letter of the alphabet is used to represent a variable.
  - ▶ Sometimes we call these letters pronumerals because they act as a numeral.
  - ▶ It is most important that the **meaning** of variables is understood.
- ▶ An **algebraic expression** might involve variables, numbers and symbols (+, −, ∞, ÷, √, ...) but *no equals sign*.
- ▶ An **equation** contains an *equals sign* (=) and indicates that two expressions are equal. Formulas are examples of equations.

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## Example of an Expression



Write an expression to represent the following situation and then simplify the expression:

Two times a number plus five times the same number.

We must first define the variable. Let the unknown number be  $x$ .

Then,

$$2 \times x + 5 \times x,$$

becomes the required expression.

Recall that it is not necessary to include the multiplication sign, so we could rewrite this expression as

$$2x + 5x.$$

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- ▶ The symbols  $+$ ,  $-$ ,  $\times$ ,  $\div$  have exactly the same meaning in algebra as in arithmetic.
- ▶ **Like terms** are those terms which contain the **same power of the same variable**.

Sort the following into groups of like terms:

$$x, 5a, 7a^2, 6x^2, 7x, 9a, 17x, 5x^2, 6a^2,$$

$$8x, 9x^2, 12a, -4x, -11x^2, -3a, -2x, 2a^2.$$

- ▶  $x, 7x, 17x, 8x, -4x, -2x$
- ▶  $5a, 9a, 12a, -3a$
- ▶  $7a^2, 6a^2, 2a^2$  and
- ▶  $6x^2, 5x^2, 9x^2, -11x^2$

## Example

Simplify

1.  $3x + 2x$ ;
2.  $5a^2 + 9a - 3a^2 - a + 1$ ;
3.  $-3x + 4y + xy + 8x - y$ .

## Solution

1.  $3x + 2x$

Are  $3x$  and  $2x$  like terms? Yes, because they have the same power of the same variable ( $x$ ). Therefore,

$$\begin{aligned} 3x + 2x &= (3 + 2)x \\ &= 5x. \end{aligned}$$

2.  $5a^2 + 9a - 3a^2 - a + 1$

See that the like terms have an  $a^2$  group, an  $a$  group and a constant group, so when collecting these together we get

$$5a^2 - 3a^2 + 9a - a + 1 = 2a^2 + 8a + 1.$$

3.  $-3x + 4y + xy + 8x - y$

See that the like terms have a  $x$  group, a  $y$  group and an  $xy$  group, so when collecting these together we get

$$8x - 3x + 4y - y + xy = 5x + 3y + xy.$$

Remember:

1. You can only add or subtract like terms.
2. If a term is just  $x$ ,  $x^2$ , etc., then the coefficient is one.
3. Take care when regrouping terms with negatives.



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