Module A2

MORE THAN JUST NUMBERS

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Introduction

Have you ever been in the situation where you:

- were the person selected to divide the bill of $126.75 between 13 people when you were all out at lunch;
- have been convinced that your telephone bill was wrong but couldn't convince the cashier;
- were unsure what your credit card statement was all about; or
- thought that your teacher had added your marks incorrectly.

This module is about more than just numbers. We start by revising many of those basic skills you completed at school (possibly some time ago) and build on these to give you more skills to confidently tackle the types of dilemmas described above.

Recently a student rang, saying:

> *I am feeling great, I recently received my bank statement for my loan account and I was able to check it myself and show the bank that they had made a mistake.*

At the completion of this module we hope that you too will have the mathematical confidence to challenge what you believe to be wrong. In the section 'A taste of things to come' we will look at some situations that you may face in everyday experience, in the workplace or in your future studies.

More formally, on successful completion of this module you should be able to:

- give examples of different types of numbers (real, rational, irrational, integers);
- represent these numbers on a number line;
- demonstrate understanding of the relationship between different types of numbers, in particular between fractions and decimals;
- perform the operations of addition, subtraction, multiplication and division on all of the above numbers;
- solve real world questions using the above numbers and techniques;
- perform complex calculations using order of operation;
- use techniques of estimation to aid in calculation; and
- use a calculator for calculations.

2.1 Our number system

The numbers and symbols that we use in mathematics are a part of our **language**. If we have a common understanding of what numbers mean and say, we can be assured that the message we want to send will be received as it was intended. When we communicate with numbers, just as with words, we have to be aware of the **purpose** of the message as well as the **audience** with whom we are communicating.

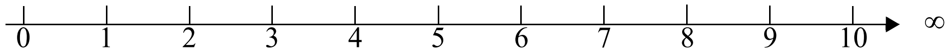
The number system we use today has developed over many hundreds of years. Early civilisations only counted using fingers and toes where necessary. When it became necessary for additional representation of number, piles of stones were used. Eventually these needed to be grouped to make counting easier. Although we now group our numbers in lots of ten, early civilisations used other groupings for their number systems. For example some grouped in lots of five because they were used to seeing five fingers and five toes on each hand and foot. Another grouping that is still in use today is 60. The Babylonian civilisations used this for counting and we continue with its use for measurement of time (60 minutes in 1 hour, 60 seconds in 1 minute).

The system of numbers we use today is called the Hindu-Arabic system. It consists of the **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 and is believed to have been used by the Arabs before being adopted by other countries. This system differed greatly from other systems because of the inclusion of a symbol for zero.

2.1.1 Whole numbers

Whole numbers are the first numbers we learn about as children. They are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, … . You probably already know many things about them.You may know that some of the numbers are **even** (2, 4, 6, 8, …) and some are **odd** (1, 3, 5, 7, …). Whole numbers form the basis for all other numbers (e.g. fractions, decimals) so our first step in this course is to take a closer look at them.

Firstly, let's picture these numbers on the **number line**. This allows us to see these numbers in order, moving from small to large. To draw a number line we draw a line and choose any point to represent zero (0). With a ruler we then mark off even spaces along the line to the right.



The arrow at the end of the line means that the numbers keep going, getting larger and larger (forever!!). We call the imaginary point that we never reach at the end of this number line, **infinity**. It is given the symbol ∞. Over history there has been much discussion about the concept of infinity. It is still a concept that is very hard to grasp. We do know that the first person to use the symbol ∞ for infinity was Englishman John Wallis in a publication in 1655.

Let's look at some features of the number line.

You know that 3 is a number less than 5 and we can represent this as $3 < 5$. The symbol '<' means **less than**.**We say** ***3 is less than 5***.

If you look at the number line above you will see that the number 3 is to the left of 5. This also indicates that $3 < 5$.

The opposite symbol to less than is **greater than** represented by the symbol '>'.

An example of this would be to say that $24 > 9$ or *24 is greater than 9*. You can imagine that 24 is further along the number line than 9 and hence is greater than 9.

You should note that the ‘point’ of the < or > sign will always point to the smaller number while the ‘open side’ will always point to the larger number.

Activity 2.1

1. Determine whether the following statements are true or false

   (a) $6 < 9$

   (b) $5 > 6$

   (c) $2 < 19$

   (d) $56 > 49$

   (e) $24 < 123$

2. Insert the correct symbol between these two numbers to make the statement true.

   (a) 6 ☐ 12

   (b) 74 ☐ 56

   (c) 127 ☐ 89

   (d) 17 ☐ 34

   (e) 41 ☐ 28

Before moving to the next section be sure to check your answers using the solutions at the end of this module.

We will look more closely at ways of comparing two or more items in a future module.

2.1.2 Place value

We call our number system the **decimal system** (‘dec’ meaning ten) because it is based on the ten **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. It is the position or place of the digit within a number that determines the value it will represent. For example, consider the 3 in the following numbers

536 and 3 598

In 536, the 3 refers to 3 tens (that is thirty) whereas in 3 598 it represents 3 thousands.

Each place value has a special name as set out in the table below.

Billions			Millions			Thousands					
hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units

Consider these numbers:

(a) three hundred and sixty two.

That is, **three** hundreds, **six** tens and **two** units.

Or 300 + 60 + 2

(b) one thousand, nine hundred and forty.

That is, **one** thousand, **nine** hundreds and **four** tens

Or 1 000 + 900 + 40

(c) eight hundred and thirty two thousand and five

That is, **eight** hundred thousands, **three** ten thousands, **two** thousands and **five** units

Or 800 000 + 30 000 + 2 000 + 5

(d) forty five

That is, **four** tens and **five** units

Or 40 + 5

(e) fifty six thousand, four hundred

That is, **five** ten thousands, **six** thousands and **four** hundreds

Or 50 000 + 6 000 + 400

(f) ninety two million, twenty four thousand and fifty six

That is, **nine** ten millions, **two** millions, **two** ten thousands, **four** thousands, **five** tens and **six** units

Or 90 000 000 + 2 000 000 + 20 000 + 4 000 + 50 + 6

Let's write them as digits in a place value table.

	Billions			Millions			Thousands					
	hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units
(a)										3	6	2
(b)									1	9	4	0
(c)							8	3	2	0	0	5
(d)											4	5
(e)								5	6	4	0	0
(f)					9	2	0	2	4	0	5	6

If we had written these numbers outside the table they would have looked like this:

(a) 362

(b) 1 940

(c) 832 005

(d) 45

(e) 56 400

(f) 92 024 056

You might have noticed with the above numbers, that when writing a number larger than 3 digits it is usual to place the digits in groups of three starting from the units column. In the past we used a comma to separate these groups. This became confusing when some countries of the world use a comma to denote the decimal point. To overcome this confusion we now separate these groups of three with a space only.

This makes the number easier to read as each group is given a name as shown at the top of the table above.

Consider the area of Australia at 7 682 300 square kilometres. We would think of this number in its three separate parts:

	7	682	300
and say	*seven*	*six hundred and*	*three hundred*
	million	***eighty two thousand***	

Try reading this number aloud 765 431 982

You should have said *seven hundred and sixty five million, four hundred and thirty one thousand, nine hundred and eighty two.*

Being able to write numbers in words is essential when writing cheques. It would be nice to see the above number written on a cheque made out to you or me!

Check out the resource CD and in particular the ‘More than just numbers’ interactive program. You will find this in the ‘Course resources’ section of the CD. When you open the ‘More than just numbers’ program, choose ‘Topics’ and work through the ‘Numbers’ topic.

2.1.3 Estimation

Have you ever been shopping and needed to check quickly that you had enough money to pay for your purchases?

When working with any numbers, but especially very large numbers, it is always a good idea to get an estimate of the answer before actually calculating. Many of our calculations will be done on a calculator and in these cases estimation will become a very important skill.

If you had $37 in your wallet you know that you would have about $40. What you are saying is that the money you have is closer to $40 than to $30.

To help us estimate answers, we have a mathematical convention for deciding how to **round off** numbers in this way.

Rounding Numbers

To round numbers to a particular place value, investigate the digit immediately to the right of it.

- If the digit immediately to the right is greater than or equal to 5 (i.e. ≥ 5, which includes the digits 5, 6, 7, 8 or 9), then increase the required place value by 1.

  We say the number has been rounded up.

- If the digit to the right is less than 5 (i.e. < 5 which includes the digits 0, 1, 2, 3 or 4), then the required place value remains the same.

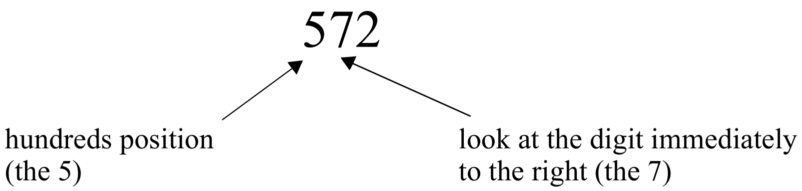
  We say the number has been rounded down.

- All digits to the right of the round off place are replaced by zeros.

Let’s look at some examples:

Example

Round 572 to the nearest hundred.

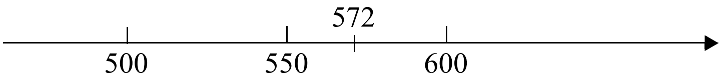


The digit immediately to the right of the hundreds column is greater than or equal to 5 so the digit in the hundreds column is increased by 1. The 5 in the hundreds place will increase to 6

All place values to the right of the required position are then filled with zeros.

So 572 rounded to the nearest hundred becomes 600.

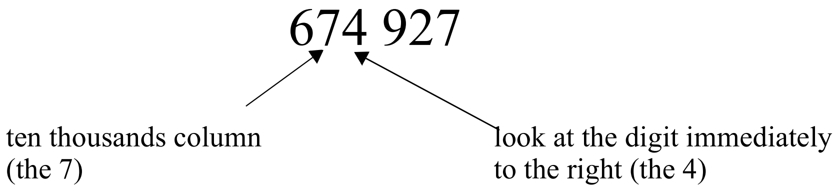
We could picture this on the number line.



We can see that 572 is closer to 600 than to 500.

Example

Round 674 927 to the nearest ten thousand.



The digit immediately to the right of the ten thousands column is less than 5 so the digit in the ten thousands column remains the same.

All place values to the right of the required position are then filled with zeros.

So 674 927 rounded to the nearest ten thousand becomes 670 000

Activity 2.2

Complete the following table by rounding each number as indicated.

	Number	To nearest ten	To nearest hundred	To nearest thousand
(a)	2 575			
(b)	324			
(c)	105			
(d)	26 897			
(e)	5 502 471			

The leading digit

Another way to round numbers that will help with estimation is to use what we call the leading digit. The **leading digit** is the first non-zero digit in a number when reading from left to right. To round to the leading digit we use the same convention as described for rounding off before, but this time our answer will have only one non-zero digit. That is, only one number that is not zero.

Example

Round 379 to the leading digit.

The first non-zero digit is 3 so we will need to round to the nearest hundred.

379 will round to 400

We can represent this in shorthand by writing

$$379 \approx 400$$

The symbol ≈ means **approximately equal to**. So we would read the above statement as *379 is approximately equal to 400.*

You may also see the symbols ≑ or ≅ used to represent ‘approximately equal to’ but in these modules we will continue to use ≈

Example

Round 23 to its leading digit.

$$23 \approx 20$$

We have rounded to the nearest ten. We would say *23 is approximately equal to 20*.

Activity 2.3

Round each number to its leading digit

(a) 32
(b) 48
(c) 137
(d) 396
(e) 223 144
(f) 881
(g) 98 101
(h) 56 135

In any calculations that follow it is always good practice to quickly estimate the answer before you calculate. Having done this you can be sure that your answer is ‘close to the mark’. We would expect you to follow this practice throughout the rest of these modules wherever possible.

2.2 Working with numbers

You will probably remember all or parts of the following from your previous studies. Although everybody should work through all sections, some people may move more quickly than others. If you are unsure, take your time, this groundwork is very important.

In mathematics there are four basic operations to perform: addition, subtraction, multiplication and division. We will deal with each of these in turn.

Although we will concentrate on using the calculator for performing these operations, the skill of doing them without a calculator is very valuable. In some courses such as nursing you will not always be permitted to use a calculator and so must do all calculations ‘by hand’. If you are going on to higher levels of mathematics you will need these calculating skills to manipulate algebraic expressions (whatever that may be!!). Even in your everyday life, you will not always have access to a calculator when you need to perform a quick calculation. For example, have you ever had to decide whether it is better to buy a packet of three items or three individually (assuming you need more than one)?

In each of the following cases we will look at how to use the calculator first and then the more traditional methods of calculating.

There are many different types of calculators and the sequence of keystrokes will vary between makes and models. An example of keystrokes is given for each case. You will need to try each case with your calculator to determine your own set of keystrokes. It is important to get to know how your calculator operates. If you have any difficulties please contact your tutor. (To contact a tutor refer to page 1 in the introductory book.

2.2.1 Addition

On a recent weekend trip I travelled the following distances each day.

Friday	Saturday	Sunday	Monday
356 km	126 km	91 km	402 km

To find the total distance travelled on this weekend trip I need to find the **sum** of the distances for each day. That is, I must **add** together each of the above distances.

Before doing any calculation we should estimate the answer. It is easy on the calculator to press the wrong key and end up with an incorrect answer. If we estimate the answer first, we should then be aware that our answer may be wrong, if the estimated and calculated answers differ by a large amount.

The most convenient way to estimate is to round each number to its leading digit.

$356 \approx 400$
$126 \approx 100$
$91 \approx 90$
$402 \approx 400$

The total distance travelled then would be approximately equal to:

$400 + 100 + 90 + 400 = 990$ km

Adding numbers on the calculator

Find the + key on your calculator.

An example of the keystrokes for finding the total distance travelled over the weekend are:

3 5 6 + 1 2 6 + 9 1 +

4 0 2 =

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 975.

This is close to our estimate of 990 so we can assume our answer is probably correct. Therefore the total distance I travelled on my weekend trip was 975 kilometres.

If you make an error when entering the numbers it is sometimes possible to correct the error without having to re-enter the whole calculation.

Find this key on your calculator. It may look like C , ◄ or DEL

If you type in an incorrect number, use this key to cancel the error and then press in the correct number and continue.

For example, if you went to press 2 + 5 but accidentally pressed 2 + 6 press the key to cancel the 6.

Now press 5 = to complete the calculation.

The display should read 7 (the sum of 2 and 5)

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Spend a couple of minutes with a calculator now. Try correcting some errors.

Adding numbers without a calculator

- The first step in this process is to **estimate** the answer by rounding to the leading digit as we did before.
- Now line up the numbers you are adding, according to their place values. Add the columns, working from **right to left**. This means that we add the smallest place values first and work through to the largest.
- As a final step **check** your answer against your estimate to see if your answer is reasonable. If you have a calculator, **verify** your answer with the calculator.

There are various methods of writing out the addition process but we will look at just one of these.

Example

Evaluate: 58 + 321

Note that evaluate in mathematics means find a single answer.

Estimate:

58 ≈ 60
321 ≈ 300
60 + 300 = 360

Calculate:

5 8	Add	Units	8 + 1 = 9
+ 3 2 1		Tens	5 + 2 = 7
3 7 9		Hundreds	3 = 3

Check: The answer is close to our estimate and so looks reasonable.

Example

From our travelling example 356 + 126 + 91 + 402

Estimate:

$$\begin{aligned} 356 &\approx 400 \\ 126 &\approx 100 \\ 91 &\approx 90 \\ 402 &\approx 400 \\ 400 + 100 + 90 + 400 &= 990 \end{aligned}$$

Calculate:

$$\begin{array}{r} 3\,5\,6 \\ 1\,2\,6 \\ 9\,1 \\ +\,4\,0\,2 \\ {\scriptstyle 1\,1\;\,} \\ \hline 9\,7\,5 \end{array}$$

Add Units 6 + 6 + 1 + 2 = 15

Write down the 5 in the units column and carry the 1 to the bottom or top of the tens column.

Tens 5 + 2 + 9 + 0 + 1 = 17

Write down the 7 in the tens column and carry the 1 to the bottom or top of the hundreds column.

Hundreds 3 + 1 + 4 + 1 = 9

Check: The answer is close to our estimate and so looks reasonable.

Activity 2.4

Find the following answers **without** using a calculator. Estimate your answer before commencing. Check your answer by using a calculator.

1.
   - (a) 58 + 61
   - (b) 25 + 956 + 32
   - (c) 750 + 2 305 + 10 + 9 + 315
   - (d) 658 + 0
2. The children's ward of the local hospital admitted 5 children on Sunday, 3 on Monday, 2 on Tuesday, none on Wednesday and Thursday but 6 on Friday and 9 on Saturday. How many children were admitted to the children's ward for the week?
3. The attendances for the Bold Batters Baseball Club's first four home games were 10 428, 8 922, 7 431 and 9 647. How many people came to the first four home games?
4. David's lunch consisted of a triple hamburger that contained 2 103 kJ (kilojoules, the measure of energy in food), hot potato chips containing 1 714 kJ, an apple pie with 1 148 kJ and a diet softdrink containing just 18 kJ. How many kilojoules did David consume for lunch?

2.2.2 Subtraction

On my previously mentioned weekend escape, I took $300. My expenses for the weekend totalled $121.

To find the amount of money I came home with I must **subtract** these expenses from the original amount of money. We call this the **difference** between the amount of money I took with me and the expenses.

That is, $300 – $121

Firstly, estimate the remaining amount.

300 = 300
121 ≈ 100

300 – 100 = 200 The remaining money is about $200

We can now do this **subtraction** on the calculator.

Subtracting numbers on the calculator

Find the − key on your calculator.

The keystrokes for finding the remaining amount of money would be:

3 0 0 − 1 2 1 =

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 179

$179 is close to our estimate of $200 so is a reasonable answer.

The amount of money I had left after the weekend escape was $179.

Subtracting numbers without a calculator

- As for addition, we must firstly estimate the answer.
- Then line up the numbers according to place value and subtract the columns working from **right to left** as for addition.
- Finally, check your answer against the estimate to see if your answer is reasonable.

Example

Evaluate 2 596 – 452

Estimate: 2 596 $\approx$ 3 000
452 $\approx$ 500
3 000 – 500 = 2 500

Calculate:

$$\begin{array}{r} 2\,5\,9\,6 \\ -\ \ 4\,5\,2 \\ \hline 2\,1\,4\,4 \end{array}$$

Subtract Units 6 – 2 = 4
Tens 9 – 5 = 4
Hundreds 5 – 4 = 1
Thousands 2 – 0 = 2

Check: The answer is close to our estimate and so looks reasonable.

This example was very simple. Most subtractions though will not be quite so straight forward. We will look at two methods of subtraction. Continue to use the method you are most familiar with. If you don't already have a method, choose the method that you find easier.

Example

Evaluate 645 – 458

Method 1

Estimate: 645 $\approx$ 600
458 $\approx$ 500
600 – 500 = 100

Calculate:

Subtract

$$\begin{array}{r} \overset{5}{\cancel{6}}\,\overset{13}{\overset{\cancel{3}}{\cancel{4}}}\,\overset{15}{\cancel{5}} \\ -\,4\,5\,8 \\ \hline 1\,8\,7 \end{array}$$

Units 5 – 8 The 8 is too large. Borrow one ten from the next column.
5 + 10 = 15 (units). One less ten leaves 3 tens.
Now, 15 – 8 = 7

Tens 3 – 5 The 5 is too large. Borrow one hundred from the next column.
3 + 10 = 13 (tens). One less hundred leaves 5 hundreds.
Now, 13 – 5 = 8

Hundreds 5 – 4 = 1

Check: The answer is close to our estimate and so looks reasonable.

Method 2

This is the ‘borrow and pay back’ method

Estimate:

$$\begin{array}{rcl} 645 & \approx & 600 \\ 458 & \approx & 500 \\ 600 - 500 & = & 100 \end{array}$$

Calculate:

Subtract Units 5 – 8 The 8 is too large. Borrow one ten from the next column.
5 + 10 = 15 (units). Now, 15 – 8 = 7
Now pay one back to the tens column.

$$\begin{array}{r} 6\ {}_{1}4\ {}_{1}5 \\ -\ 4_{1}\ 5_{1}\ 8 \\ \hline 1\ \ 8\ \ 7 \end{array}$$

Tens 4 – 6 The 6 is too large. Borrow one hundred from the next column.
4 + 10 = 14 (tens). Now, 14 – 6 = 8
Now pay one back to the hundreds column.

Hundreds 6 – 5 = 1

Check: The answer is close to our estimate and so looks reasonable.

Activity 2.5

Find the following answers **without** using a calculator. Estimate your answers before you commence. Check your answers with a calculator.

1.
   (a) 759 – 326

   (b) 39 200 – 14 125

   (c) 126 430 – 2 472

   (d) 37 – 0

2. Mary purchases 4 items costing $14, $27, $16 and $7. She pays for the items with a $100 note. How much change should Mary receive?

3. A cleaner is asked to dilute 542 mL of disinfectant concentrate by making it into 3 500 mL of diluted solution. How much distilled water will the cleaner need to add? A diagram may help you ‘picture’ this situation.

4. The following table shows the weekly pay schedule for a number of employees of a small company. Gross pay is the amount of money you receive before any deductions. Deductions for these employees include taxation, superannuation and union fees. After these deductions have been made, the amount remaining is the take home pay.

Name	**Gross pay**	**Tax**	**Superannuation**	**Union fees**	**Take home pay**
Adams J	$500	$105	$30	$2	
Bull P	$1 200	$407	$74	$2	
Filbee Y	$678	$169	$41	$2	
Hand I	$893	$261	$54	$2	
Ruse K	$560	$127	$34	$2	
Totals					

(a) For each employee calculate the take home pay.

(b) What is the total wages bill for these employees (Gross Pay)?

(c) What amount of money will be sent to the taxation office on behalf of these employees?

5. The following table shows the Fly Buys points needed for return flights between these cities.

Points for return flights

	Mystery flight	Adelaide	Alice Springs	Brisbane	Cairns	Canberra	Gold Coast	Hobart	Launceston	Melbourne	Perth	Sydney	Townsville
Adelaide	550		1650	1850	2100	1350	1800	1400	1350	850	1950	1250	2100
Brisbane	550	1850	2300		1450	1350		1800	1800	1450	2800	850	1350
Canberra	550	1350	2100	1350	2200		1400	1500	1450	800	2450	650	2050
Darwin		2350	1550	2650	1900	2650	3000	2750	2750	2750	2750	2750	2950
Gold Coast		1800	2350		1450	1400		1800	1800	1400	2850	850	1450
Hobart		1400	2250	1800	2600	1500	1800			850	2400	1200	2400
Launceston		1350	2200	1800	2600	1450	1800			800	2400	1200	2400
Melbourne	550	850	2200	1450	2300	800	1400	850	800		2050	850	2150
Perth	550	1950	2050	2800	2550	2450	2850	2400	2400	2050		2350	3300
Sydney	550	1250	2200	850	1900	650	850	1200	1200	850	2350		1850
Townsville		2100	1850	1350	700	2050	1450	2400	2400	2150	3300	1850	

(All points shown are for single return flights)

(a) How many points do I need to take a return trip from the Gold Coast to Townsville?

(b) Linda decides to fly from Brisbane to Sydney. How many points does she need for the return trip?

(c) While still in Sydney, before returning to Brisbane, Linda decides that she will take a return flight from Sydney to Melbourne. How many points does she need for this return trip?

(d) How many points would Linda have saved by flying directly from Brisbane to Melbourne. (Assuming she did not want to visit Sydney of course!)

Subtraction that gives a negative answer

Consider the following.

The temperature in Toowoomba on a winters evening is 8°C. By midnight the temperature has dropped by 5°C.

What will the temperature be at midnight?

....

Did you say 3°C? We might think of the subtraction 8°C – 5°C.

What if the temperature drops a further 5°C by 4 a.m. What will the temperature be at 4 a.m.?

....

You may have come up with many different ways to express this answer. The following are some of the possible answers:

2 degrees less than zero
2 degrees below zero
negative two degrees

or maybe – 2°C

If we think of this as a subtraction we would write

3°C – 5°C

If you do the subtraction 3 – 5 on the calculator, the display should read –2.

In general when subtracting two numbers, if the number being subtracted is the larger of the two the answer will be negative.

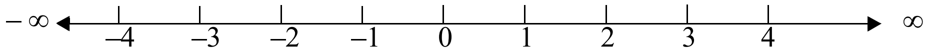
Which of the following will give negative answers?

(a) 243 – 672
(b) 2 430 – 1 650
(c) 246 – 1 240
(d) 21 583 – 9 683

You should have said that (a) and (c) would give negative answers.

It is clear that the whole numbers we originally looked at are no longer enough to cope with the needs of our daily calculations. We must extend these numbers below zero to include the **negative** numbers.

Let's picture this on the number line.



We call the positive and negative whole numbers together with zero, the **integers**.

We can see that –2 is to the left of 6 on the number line so we can say that *–2 is less than 6*. We write this –2 < 6.

We can also see that –5 is to the left of –1 so we can say that *–5 is less than –1.* We write this –5 < –1. If you think of this as temperature, –5°C is much colder than –1°C and so –5 is less than –1.

Activity 2.6

Insert the symbol that will make the following statements true.

(a) –5 ☐ 3

(b) –1 ☐ –4

(c) 3 ☐ –1

(d) –65 ☐ –34

(e) –125 ☐ –420

We will look more closely at working and calculating with negative numbers in a later section of this module.

2.2.3 Multiplication

Suppose a builder is prepared to employ you as a casual labourer for \$80 a day.

Your total weekly pay would be: $\underbrace{\$80 + \$80 + \$80 + \$80 + \$80}_{\text{5 lots of \$80 added together}} = \$400$

A shorthand way to write 5 lots of \$80 added together is to use **multiplication**.

That is, $5 \times 80 = 400$

We say ***five multiplied by eighty equals four hundred***.

We call 5 and 80 **factors** of 400 and say that 400 is the **product** of 5 and 80.

You may be familiar with these terms in everyday language. For example, 'one of the **factors** leading to an early arrest was the accurate description given to police by the witness'. This simply means a part of the reason for the early arrest was the good description. We could also say 'the reason for an early arrest was a **product** of good police work and an observant witness'. We are saying that the early arrest was a combination of these two factors.

Multiplying numbers on the calculator

Find the X key on your calculator

To calculate 5 × 80, press

5 X 8 0 =

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 400

Multiplying numbers without a calculator

Before moving on to look at multiplying numbers without a calculator it is important to be able to multiply the single digit numbers. In fact it is important to know the multiplication tables up to 10 and even 12 if possible. You will need to know these basic multiplications before you can move on to multiply and divide numbers with more than one digit. To help you remember the multiplication tables you could write them out and stick them in a spot where they will be seen regularly (e.g. the fridge, above your desk, or even the toilet door!). You could also recite them onto a tape and play them back when you are doing some mundane task (driving, sleeping or dare I say working?)

Complete the following table if you are not sure of the multiplication tables (use your calculator if you need to). Keep it for reference until you become familiar with these basic products.

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0												
2	0												
3	0												
4	0												
5	0									45			
6	0												
7	0												
8	0												
9	0					45							
10	0												
11	0												
12	0												144

You should notice from the table that zero multiplied by anything is zero.

3 × 0 = 0, 7 × 0 = 0, 0 × 5 = 0, 0 × 3 = 0

To think of this in a practical situation, we could have three people each receiving zero pieces of cake 3 × 0. Altogether we still have no pieces of cake 3 × 0 = 0.

Also, think of purchasing 5 items for 9 cents or 9 items for 5 cents. Either way the cost is still 45 cents. That is, 5 × 9 = 45 and 9 × 5 = 45 so you only really need to learn about half the table.

Let's now look at multiplying larger numbers together without a calculator.

- The first step as always is to estimate the answer.
- To multiply two whole numbers line them up according to their place values. Multiply the numbers working from **right to left**. Add up the separate results to reach a final answer.
- Check this answer against your estimate.

Example

Evaluate 42 × 6

Estimate:

$$\begin{aligned} 42 &\approx 40 \\ 6 &= 6 \\ 40 \times 6 &= 240 \end{aligned}$$

Multiply the 4 and 6 and add one zero to the answer (because there was one zero in the question)

Calculate:

$$\begin{array}{rrr} & 4 & 2 \\ & \times & 6 \\ \hline & {}^{1} & \\ 2 & 4 & 2 \\ \hline 2 & 5 & 2 \end{array}$$

Multiply

Units

6 × 2 = 12

Write down the 2 in the units column and carry the 1 to the tens column of the answer.

6 × 4 = 24

Write down the 4 in the tens column and the 2 in the hundreds column of the answer.

Add

Units = 2

Tens 1 + 4 = 5

Hundreds = 2

Check: The answer is close to our estimate and so looks reasonable.

Example

Evaluate 847 × 36

Estimate:

$$\begin{aligned} 847 &\approx 800 \\ 36 &\approx 40 \\ 800 \times 40 &= 32\,000 \end{aligned}$$

Multiply the 8 and 4 and add three zeros to the answer (because there were three zeros in the question)

Calculate:

$$\begin{array}{rrrrr} & & 8 & 4 & 7 \\ & & \times & 3 & 6 \\ \hline & & {}^{2} & {}^{4} & \\ & 4 & 8 & 4 & 2 \\ & {}^{1} & {}^{2} & & \\ 2 & 4 & 2 & 1 & 0 \\ {}^{1} & {}^{1} & & & \\ \hline 3 & 0 & 4 & 9 & 2 \end{array}$$

Multiply

Units

6 × 7 = 42 Write down the 2 in the units column and carry the 4 to the tens column of the answer.

6 × 4 = 24 Write down the 4 in the tens column and carry the 2 to the hundreds column of the answer.

6 × 8 = 48 Write down the 8 in the hundreds column and the 4 in the thousands column of the answer.

Tens

Write down a zero in the units column to show that we are really multiplying by 30 not 3

3 × 7 = 21 Write down the 1 in the tens column and carry the 2 to the hundreds column of the answer.

3 × 4 = 12 Write down the 2 in the hundreds column and carry the 1 to the thousands column of the answer.

3 × 8 = 24 Write down the 4 in the thousands column and the 2 in the ten thousands column of the answer.

Add

Units 2 + 0 = 2

Tens 4 + 4 + 1 = 9

Hundreds 2 + 8 + 2 + 2 = 14 Write down the 4 and carry the 1

Thousands 4 + 1 + 4 + 1 = 10 Write down the 0 and carry the 1

Ten Thousands 2 + 1 = 3

Check: The answer is close to our estimate and so looks reasonable.

Activity 2.7

Find the following answers **without** using a calculator. Estimate your answers before you commence. Check your answer with a calculator.

1.
   - (a) 9 × 45
   - (b) 93 × 72
   - (c) 195 × 24
   - (d) 1 952 × 346
   - (e) 589 × 40

2. Sally earns $14 per hour and works for 38 hours in a week. How much does Sally earn?

3. Philip and Denise wish to pave an area around their swimming pool. They have calculated that they need 36 concrete blocks each weighing 22 kilograms. What is the total weight of concrete that Philip and Denise will need?

4. Jeremy sees a shirt normally priced at $45 on sale for only $26.
   - (a) How much would Jeremy save if he purchased a shirt at the sale price?
   - (b) Jeremy decides to buy 5 shirts in different colours. How much does he save by buying them all at the sale price?

5. An express coach from Brisbane to Melbourne stops to load and unload passengers only in Sydney.

	Number of passengers	
City	Loaded	Unloaded
Brisbane	36	
Sydney	23	14
Melbourne		

   - (a) How many passengers travelled all the way from Brisbane to Melbourne?
   - (b) How many passengers got off the coach in Melbourne?

Power notation

Often you will see different symbols being used as ways to reduce the writing out of a long process. So far we have used multiplication as a shorthand way of writing repeated additions. You will come across more of these abbreviations throughout these modules.

Power notation is a shorthand way of representing the same number being multiplied together several times.

Consider 6 × 6. This could be written 6^2. We would say *six squared* or *six to the power two.*

6^2 indicates that we are multiplying together two numbers which are both 6.

6 × 6 × 6 would be written 6^3 and would mean that we are multiplying three numbers together which are all 6. We say *six to the power three* or *six cubed.*

See if you can write out and describe in words what is meant by 6^5

……………………………………………………………………………………………………

Did you say something like 6^5 means 5 numbers which are all 6 are multiplied together to give 6 × 6 × 6 × 6 × 6?

We will look more closely at power notation in a later module but for now let's see how to evaluate powers on your calculator.

To evaluate powers on the calculator

There are two buttons on your calculator that we can use to evaluate powers

Find the $\boldsymbol{x}^2$ on the $\boxed{x^2}$ key

and the $\boldsymbol{x}^{\boldsymbol{y}}$ on the $\boxed{y^x}$ key

$\boxed{x^2}$ key is only for finding powers of 2.

To evaluate 6^2 press $\boxed{6}$ $\boxed{x^2}$ $\boxed{=}$

The display should read 36.

Depending on the type of calculator it may or may not be necessary to press the = key.

This key is very useful when finding powers of two but the key for finding all powers including 2 is the $\boldsymbol{x}^{\boldsymbol{y}}$ key.

An example of keystrokes to evaluate 5^3 are $\boxed{5}$ $\boxed{y^x}$ $\boxed{3}$ $\boxed{=}$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 125

Remember that all calculators are different. Some keys are written on the face of the calculator rather than on the keys themselves. To activate any of the calculator functions in this position it may be necessary to press another function key first. This key may be a shift key SHIFT or 2nd Function key 2nd F. For example if the key to evaluate powers looks like s.x X the sequence of keystrokes for 5^3 will be SHIFT s.x X =

You may not have had to use these two new keys before so here is an activity if you feel you would like more practice.

Activity 2.8

1. Evaluate the following without a calculator. Check your answers with your calculator.
   (a) 3^3
   (b) 2^4
   (c) 5^4
   (d) 4^1
   (e) 9^2
   (f) 6^5
2. Complete:
   (a) $4^{\square} = 16$ (How many times is 4 multiplied together to give 16?)
   (b) $2^{\square} = 32$
   (c) $5^{\square} = 25$
   (d) $3^{\square} = 81$
3. The Heptane family has seven children. Each of the seven children spends 7 minutes per day reading. What is the total time the children spend on reading in one week?

The square root

What if a question on powers asked you to find a **positive number** that when raised to the power 2 gave you 49 as the answer.

That is $\boxed{}^{2} = 49$

Did you get 7 for this answer? Check this with your multiplication table.

We have a shorthand way to express this type of question.

We could write $\sqrt{49}$ and it is understood that we are meant to find the two identical numbers that multiply to give 49.

We would read $\sqrt{49}$ as *the square root of 49*

The symbol we use for the **square root** is $\sqrt{}$

To evaluate square roots on the calculator

Find the √ key on your calculator

To evaluate $\sqrt{49}$ press

√ 4 9 =

The display should read 7

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Activity 2.9

Evaluate the following. Check your answers on the calculator.

(a) $\sqrt{81}$ Remember, we are asking what two identical numbers multiply to give 81.

(b) $\sqrt{36}$

(c) $\sqrt{25}$

(d) $\sqrt{64}$

(e) $\sqrt{100}$

2.2.4 Division

Recall that multiplication is a shorthand way of adding the same number many times. Similarly, division is a shorthand way of subtracting the same number several times.

For example, to determine how many 5's are contained in the number 20, you could use subtraction as follows:

20 – 5	=	15	one step
15 – 5	=	10	two steps
10 – 5	=	5	three steps
5 – 5	=	0	four steps

Using subtraction it has taken **four** steps to determine how many 5's there are in 20. A shorthand way of writing this using division would be:

$$20 \div 5 = 4$$

We would say this as *twenty divided by five equals four*.

We call the result of division a **quotient.** In our example above the quotient is 4.

It is interesting to note that the word quotient comes from the Latin word *quotiens* meaning 'how often' or 'how many times'.

Dividing numbers on the calculator.

Find the ÷ key on your calculator

To divide 20 by 5 press 2 0 ÷ 5 =

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 4

We can say that 20 ÷ 5 = 4 because there are 4 lots of 5 in 20 i.e. 4 × 5 = 20

Can you see that division is the opposite of multiplication. Knowing your multiplication tables will be a help when doing division as well as multiplication.

For example:

72 ÷ 8	=	9	since	9 × 8	=	72
45 ÷ 9	=	5	since	5 × 9	=	45

Look at the multiplication tables you wrote out earlier and use them as a division table.

Before we move on to the next section let’s consider divisions involving zero.

$0 \div 6 = 0$ since $0 \times 6 = 0$ No cake divided among 6 children!

$0 \div 342 = 0$ since $0 \times 342 = 0$

What happens when it is zero that we are dividing by?

For example, $7 \div 0 = ?$ since $? \times 0 = 7$

We know that anything multiplied by zero equals zero so there is **no** number that we can write instead of the ? sign.

We say that division by zero is undefined. That is, it is impossible to divide by zero.

Try 7 ÷ 0 on your calculator. You should get an error message (- *E* -)

Dividing numbers without the calculator

- The first step in this process will be to estimate the answer.
- Then divide the numbers, working from **left to right** this time. That is from the highest place value to the lowest.
- Finally check for the reasonableness of your answer.

Example

Evaluate $864 \div 6$

Estimate:

$864 \approx 900$

$6 = 6$

$900 \div 6$ 6 goes into 9 more than once but not twice.
The answer will be between 100 and 200

Calculate:

We will show this process as several steps. You would normally do all this on one diagram.

Divide

$$\begin{array}{rccc} & 1 & & \\ \cline{2-4} 6\big) & 8 & 6 & 4 \\ & \underline{6} & & \\ & 2 & 6 & \end{array}$$

How many 6’s are there in 8? Answer 1. Write the 1 above the 8 and multiply the 1 by the 6 and write this answer below the 8. Subtract the 6 from the 8. Bring down the 6 from beside the 8.

$$\begin{array}{rccc} & 1 & 4 & \\ \cline{2-4} 6\big) & 8 & 6 & 4 \\ & \underline{6} & & \\ & 2 & 6 & \\ & 2 & 4 & \\ \cline{2-3} & & 2 & 4 \end{array}$$

How many 6’s are there in 26? Answer 4. Write the 4 above the 6 in the question and multiply the 4 by the 6 (24) and write this answer below the 26. Subtract the 24 from the 26. Bring down the 4 from beside the 6.

$$\begin{array}{r|lll} & 1 & 4 & 4 \\ \cline{2-4} 6 & 8 & 6 & 4 \\ & \underline{6} & & \\ & 2 & 6 & \\ & \underline{2} & \underline{4} & \\ & & 2 & 4 \\ & & \underline{2} & \underline{4} \\ & & & 0 \end{array}$$

How many 6's are there in 24? Answer 4. Write the 4 above the 4 in the question and multiply the 4 by the 6 (24) and write this answer below the 24. Subtract the 24 from the 24.

Our answer is telling us that there are exactly 144 lots of 6 in 864.

Check: The answer is within the range of our estimate so looks reasonable.

Example

Evaluate $3\,854 \div 41$

Estimate:

$$\begin{aligned} 3\,854 &\approx 4\,000 \\ 41 &\approx 40 \\ 4\,000 \div 40 &= 100 \end{aligned}$$

Calculate:

Divide

$$\begin{array}{r|llll} & & & 9 & 4 \\ \cline{2-5} 41 & 3 & 8 & 5 & 4 \\ & \underline{3} & \underline{6} & \underline{9} & \\ & & 1 & 6 & 4 \\ & & \underline{1} & \underline{6} & \underline{4} \\ & & & & 0 \end{array}$$

41 will not divide into 3 so try 41 into 38. This won't go either so try 41 into 385. This goes about 9 times. Write the 9 above the 5 and multiply the 9 by the 41(369) and write this answer below the 385.
Subtract the 369 from the 385. Bring down the 4 from beside the 5.
How many 41's are there in 164? Answer 4. Write the 4 above the 4 in the question and multiply the 4 by the 41 (164) and write this answer below the 164. Subtract the 164 from the 164.

Check: The answer is close to our estimate so looks reasonable.

Check out the resource CD and in particular the intractive program 'More than just numbers'. You will find this in the 'Course resources' section of the CD. Select 'Topics' and work through the 'Calculations' topic.

Activity 2.10

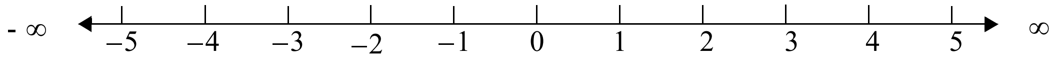
Complete the following **without** using a calculator. Estimate your answer before you begin. Check your answer with the calculator.

1.
   - (a) 1 204 ÷ 4
   - (b) 432 ÷ 12
   - (c) 10 608 ÷ 26
   - (d) 3 612 ÷ 42
2. Three children were left a $4 500 inheritance. How much will each child receive?
3. Joseph purchased 2 700 grams of minced steak to be used for 6 family meals. He wishes to freeze the meat in meal size portions. What amount of mince should he use for each meal?
4. If you receive an annual salary of $31 025, what is your weekly wage? Hint: you will need to work out the daily rate (365 days in a year) and then the weekly rate.
5. A merry-go-round at the local show revolves once every 32 seconds. Keely's ride lasts 8 minutes, how many times did the merry-go-round revolve? (Hint: 8 minutes equals 8 × 60 = 480 seconds).

2.3 Calculations involving negative numbers

Earlier in this chapter we showed how our number system has been extended to include the positive and negative whole numbers and zero. So far we have looked at performing the four basic operations on positive numbers. Let's now take a look at calculating with the negative numbers.

Recall the number line



You should check that the distances between adjacent (lying beside each other) integers on the number line are always the same.

Let's look more closely at the temperature example that generated this need for an extended number system.

Recall:
Module 2
Section 2.2.2

Recall that at 4 am on that chilly Toowoomba morning the temperature was −2 degrees. Suppose that two hours later the temperature had risen by 3 degrees.

What was the temperature at 6 am? You may find the number line above will help.

....

Did you say something like 1 degree?

If you were asked to calculate −2 + 3 what would your answer be? Explain why?

....

Although this question is represented by symbols, you could think of it in exactly the same way as you did the temperature change. On the number line you begin at −2 and move three ‘jumps’ to the right to indicate an increase. Your answer should have been 1 as before. We can write −2 + 3 = 1

By 6.30 am the temperature had risen to 3 degrees (1 + 2 = 3) but unfortunately half an hour later it had dropped by 5 degrees.
What was the temperature at 7 am?

....

Did you say negative 2 degrees?

If you were asked to calculate 3 − 5 what would your answer be? Explain why?

....

Again you can think of this as a temperature question and start at 3 on the number line and move 5 ‘jumps’ to the left to indicate a decrease. You should get −2 as before. We can write 3 − 5 = −2. We can also write 3 + $^{-}5$ = −2 to indicate we started at 3 and added a decrease of 5.

After a cold day the temperature at 11 pm that evening was −4 degrees and by midnight it had dropped a further 3 degrees.
What was the temperature at midnight?

....

This situation may be a little more unfamiliar to you. Did you come up with −7 degrees as your answer?

If you were asked to calculate −4 – 3 what would your answer be?

....

Although this is in symbols now it is still the same question as before. On the number line start at −4 and move 3 ‘jumps’ to the left to indicate a decrease. You should end up at −7. We can write −4 – 3 = −7.

Is this the same as −4 + $^{-}3$? Explain why?

....

Yes it is the same. In our temperature example we could think of this as adding a fall in temperature, that is -3 degrees. Subtracting a number is the same as adding the negative of the same number. We will look more closely at this in the next section.

You should now feel a little more comfortable with the concept of a negative number. You may be interested to know that the word negative comes from the Latin ***negatus*** which means to deny. Negative numbers were thus named since many people denied they had any real meaning. Even mathematicians in the 15th and 16th centuries thought them absurd or impossible answers.

We have just shown that they do have meaning. Negative numbers are not something to be afraid of, but rather a part of the natural order of things. Of course if you live in more tropical climates than Toowoomba it may not be temperature where these numbers occur in your daily life. Perhaps it is going above or below sea level or in and out of debt.

2.3.1 Addition involving negative numbers

Consider the cases we have looked at with our temperature example.

$$\begin{aligned} -2 + 3 &= 1 \qquad (1) \\ 1 + 2 &= 3 \qquad (2) \\ 3 + {}^{-}5 &= -2 \qquad (3) \\ -4 + {}^{-}3 &= -7 \qquad (4) \end{aligned}$$

In examples (2) and (4) you can see that we are adding together numbers with the same sign, that is two positive numbers and two negative numbers.

In both cases the answer has the same sign as the numbers being added.

Adding two positive numbers gives us a larger positive number. We have moved to the right on the number line.

Adding two negative numbers gives us a larger negative number. We have moved to the left on the number line.

That is $-4 + {}^{-}3 = -(4 + 3) = -7$

The brackets indicate that we are grouping the numbers together after the negative sign. We should work out the brackets and then attach the negative sign.

This makes these types of addition easy.

Example

$-3 + {}^{-}12$

The signs are the same so our answer will also be negative.

$-3 + {}^{-}12 = -(3 + 12) = -15$

Negative numbers on the calculator

Find the (−) key on your calculator. It may also look like +/−

To calculate $-3 + {}^{-}12$ press (−) 3 + (−) 1 2 =

If you are using +/− key the sign is given after the number. For example

3 +/− + 1 2 +/− =

The display should read – 15

Did you notice that when you pressed the (−) or +/− key a negative sign appeared before the 3 on the calculator display? This key changes the sign of whatever is on the display of the calculator.

Here are some for you to try.

Activity

Activity 2.11

1. Evaluate the following without a calculator. Check your answers with a calculator.
   (a) $-7 + {}^{-}16$
   (b) $-4 + {}^{-}9$
   (c) $-25 + {}^{-}12$
   (d) $-456 + {}^{-}32$
   (e) $-3 + {}^{-}1\ 245$

   Translate each of the following questions into an expression involving a sum and then solve.
2. A submarine is 10 metres below sea level. It dives a further 20 metres. How far below sea level does it come to rest?
3. Dan’s cheque account is over-drawn by $110. If he writes another cheque for $43, what will Dan’s cheque account balance now show?

Let’s return to the other two types of addition that we have come across.

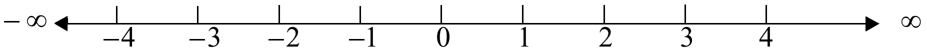
$$\begin{aligned} -2 + 3 &= 1 && (1) \\ 3 + {}^{-}5 &= -2 && (3) \end{aligned}$$

In these cases the signs of the terms being added are different. As you can see from these examples, sometimes the answer will be positive, sometimes negative.

How do you tell?

Look at the two numbers you are adding together. Which number is furthest away from the zero on the number line? It is the sign of this number that will be the sign of the answer.

Consider –2 + 3.



Now, –2 is 2 units away from zero while 3 is three units away from the zero. Since the 3 is the furthest away, the answer will be positive.

To calculate the answer, ignore the signs, take the smaller number from the larger and give the answer the sign of the larger. Let's look at this for the following:

Example

Evaluate –15 + 3 =

The –15 is further from zero than the 3 on the number line and since this number is negative, the answer will be negative.

Ignoring the signs, take the smaller from the larger.

15 – 3 = 12

Now give the answer the correct sign, that is, the answer is –12.

–15 + 3 = –12

Check on the calculator. Press (–) 1 5 + 3

Let's try another.

Example

Evaluate 78 + ⁻42 =

Firstly decide on the sign of the answer ....

Did you decide the answer would be positive in this case?

Ignoring the signs, take the smaller from the larger

78 – 42 = 36

Now write out the answer

78 + ⁻42 = 36

Check on the calculator. Press 7 8 + (–) 4 2 =

Activity 2.12

1. Complete these **without** a calculator and then check your answers with the calculator.
   - (a) $-5 + 3$
   - (b) $-56 + 78$
   - (c) $456 + {}^{-}67$
   - (d) $89 + {}^{-}567$
   - (e) $-1\,789 + 1\,674$

Translate each of the following questions into an expression involving a sum and then solve. A diagram may sometimes help you to estimate whether the answer will be positive or negative.

2. Nami owes \$25. If she is able to pay back \$5, how much does she still owe?
3. A submarine dived 37 metres and then rose 23 metres. What is its new depth?
4. Australia started their second innings in the cricket, 134 runs behind South Africa.
   - (a) If the opening partnership scored 76 runs, what is Australia’s position?
   - (b) If Australia scored 475 runs in their second innings, what is their position at the end of the innings?
5. From a certain floor in a building the lift descended 3 floors. It then rose 6 floors before descending 8 floors. What is the lift’s final position relative to its starting position?

2.3.2 Subtraction involving negative numbers

Let’s firstly consider the following examples from the temperature exercise earlier in this module. In both of these cases we are subtracting a positive number.

$$\begin{aligned} -4 - 3 &= -7 \\ 3 - 5 &= -2 \end{aligned}$$

We can write both of these as additions as we saw in the previous section and follow the procedures for adding numbers involving negatives.

That is, $-4 - 3 = -7$ becomes $-4 + {}^{-}3 = -7$

and $3 - 5 = -2$ becomes $3 + {}^{-}5 = -2$

Now let’s look at examples involving subtracting negative numbers.

$$\begin{aligned} 3 - {}^{-}5 \\ -4 - {}^{-}3 \end{aligned}$$

Consider the following statements that you might sometimes hear children make. (Having heard this, we would of course encourage better English expression!)

'I did nothing'	This is a **negative** statement. The person is saying that they didn't do anything.
'I didn't do nothing'	This statement on the other hand is **positive**. The person is saying the opposite of the first statement because of the second negative, 'didn't'. As the person didn't do nothing they must have done something.

In the second statement the person has used a double negative and arrived at a positive. We use this same principle in mathematics. Two negatives together make a positive. *(Two wrongs don't make a right. But here in maths we think they might!!!)*

Therefore we can write the above two statements as

$$\begin{aligned} 3 - {}^{-}5 &= 3 + 5 = 8 \\ -4 - {}^{-}3 &= -4 + 3 = -1 \end{aligned}$$

We have then reduced all subtractions to additions so we need only remember the addition rules.

Example

Evaluate	$-7 - 5$	Change to an addition
	$= -7 + {}^{-}5$	Same signs so answer will be negative
	$= -(7 + 5)$	
	$= -12$	

☑ Now try this on your calculator.

Example

Evaluate	$-7 - {}^{-}5$	Change to an addition
	$= -7 + 5$	Different signs. Decide that answer will be negative because –7 is further from zero than 5
	$= -(7 - 5)$	Ignoring signs take the smaller number from the larger.
	$= -2$	

☑ Now try this on your calculator.

Let's look at some examples using larger numbers.

Example

Evaluate	$-127-56$	Change to an addition
	$=-127+{}^{-}56$	Same signs so answer will be negative
	$=-(127+56)$	
	$=-183$	

☑ Now try this on your calculator.

Example

Evaluate	$82-178$	Change to an addition
	$=82+{}^{-}178$	Different signs. Decide that answer will be negative because −178 is further from zero than 82
	$=-(178-82)$	Ignoring signs take the smaller number from the larger.
	$=-96$	

☑ Now try this on your calculator.

Example

Evaluate	$-254-{}^{-}897$	Change to an addition
	$=-254+897$	Different signs. Decide that answer will be positive because 897 is further from zero than −254
	$=+(897-254)$	Ignoring signs take the smaller number from the larger.
	$=+643$	
	$=643$	It is not necessary to write in the + sign.

☑ Now try this on your calculator.

Activity 2.13

1. Find the following **without** a calculator. Check your answers using the calculator.

(a) –45 – 89
(b) 456 – 765
(c) –376 – –46
(d) 73 – –34
(e) 567 – 345
(f) –498 – –587
(g) –45 – 238
(h) 1 234 – –890
(i) –56 – 4 693
(j) 567 – –780

2. Complete the following without a calculator.

(a) 13 – ☐ = 5
(b) 7 – ☐ = –2
(c) –3 – ☐ = –6
(d) 7 – ☐ = 8
(e) ☐ – 3 = –2
(f) ☐ – 3 = –9

Translate each of the following questions into an expression involving a difference and then solve.

3. For the following cities calculate the **difference** in temperature between the maximum and minimum temperatures.

City	**Maximum °C**	**Minimum °C**	**Difference °C**
London	11	8	
Moscow	8	–1	
New York	3	–4	
Helsinki	–2	–6	

4. A man dived into a lake from the top of a 20 metre cliff. He levelled out after dropping 34 metres. At what depth did he level out?

5. At the end of a game Bill had scored 23 points while Peter had scored –4 points. By how many points did Bill beat Peter?

2.3.3 Multiplication involving negative numbers

Recall that multiplication can be thought of as a shorthand way of writing many additions of the same number. For example:

$4 + 4 + 4 = 12$ we can write as $4 \times 3 = 12$

Negative numbers work in the same way

$-4 + {}^{-}4 + {}^{-}4 = -12$ we can write as $-4 \times 3 = -12$

We can apply this finding as a general rule.

When multiplying numbers of opposite sign (one positive and one negative) the answer will be negative.

Another rule that you must know is for the case when multiplying numbers of the same sign. This rule is not easily explained by even the most brilliant mathematician so we give it to you without proof.

When multiplying numbers of the same sign (both positive or both negative) the answer will be positive.

An example of this would be:

$-6 \times {}^{-}5 = 30$

In general when multiplying numbers involving negatives, decide on the sign of the answer using the above rules. Then ignore the signs and perform the multiplication. Finally give your answer the sign you determined in the first step.

Let's put all this into practice.

Example

Evaluate $42 \times {}^{-}23$

Look at the question and determine that the answer will be negative because the signs of the numbers are different.

Now ignore the signs and multiply

$42 \times 23 = 966$

estimate:
$$\begin{aligned} 42 &\approx 40 \\ 23 &\approx 20 \\ 40 \times 20 &= 800 \end{aligned}$$

This answer appears reasonable.

Finally attach the sign you determined in the first step

$42 \times {}^{-}23 = -966$

Before we move on with some for you to try, let's look at doing this question on the calculator.

To evaluate 42 × −23 on the calculator, press the following keys:

4 2 × (−) 2 3 =

or

4 2 × 2 3 +/− =

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read −966 as you would expect.

Activity 2.14

1. Evaluate the following without a calculator. Where appropriate, estimate your answer before you begin. Check your answers on a calculator.

(a)	−3 × 5	(f)	−34 × 29
(b)	4 × −7	(g)	37 × −87
(c)	−6 × −8	(h)	−104 × 6
(d)	−12 × 6	(i)	789 × −45
(e)	−6 × 0	(j)	−67 × −45

Translate each of the following questions into an expression involving a product and then solve.

2. The temperature was –3° yesterday. If it is twice as cold today, what is the temperature today?
3. A diver was 2 metres below sea level. The sea bed was 5 times deeper than his position. At what depth is the sea bed?

2.3.4 Division involving negative numbers

The same rules that we applied for multiplication involving negative numbers will apply for division.

That is:

When dividing numbers of opposite sign (one positive and one negative) the answer will be negative.

and

When dividing numbers of the same sign (both positive or both negative) the answer will be positive.

In general, when dividing numbers involving negatives, decide on the sign of the answer using the above rules, then ignore the signs and perform the division. Finally give your answer the sign you determined in the first step.

Let's look at an example.

Example

Evaluate $-45 \div 5$

Look at the question and determine that the answer will be negative because the signs of the numbers are different.

Now ignore the signs and divide

$45 \div 5 = 9$ estimate: $45 \approx 50$

$50 \div 5 = 10$

The answer look reasonable.

Finally attach the sign you determined in the first step

$-45 \div 5 = -9$

☑ Now try this on your calculator.

Activity

Activity 2.15

1. Evaluate the following. Estimate your answer, where necessary, before you begin.

   (a) $-42 \div 6$

   (b) $64 \div -8$

   (c) $-861 \div -3$

   (d) $-12 \div 6$

   (e) $-6 \div 2$

   (f) $-32 \div 16$

   (g) $5\ 642 \div -91$

   (h) $-5\ 688 \div 6$

   (i) $765 \div -45$

   (j) $-135 \div -45$

Check out the resource CD and in particular the 'More than just numbers' interactive program. You will find this in the 'Course resources' section of the CD. When you open the 'More than just numbers' program, choose 'Topics' and work through the 'Negative numbers' topic.

2.4 Order of calculation

So far we have been dealing with expressions involving just one operation, for example:

$$-12 \times 6 =$$

However, in reality, calculations usually involve performing more than one operation.

Consider the following situation. We are asked to calculate the amount earned by a person working for 12 hours if the pay rate is \$14 per hour for the first 8 hours and \$17 for each hour after that.

This means that the person is working 8 hours at \$14 per hour and 4 hours at \$17 per hour. We can express this mathematically as:

$$8 \times \$14 + 4 \times \$17$$

Look carefully at this expression. Note that there are **three** operations: a multiplication, an addition and another multiplication.

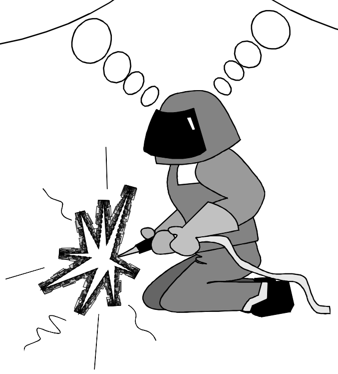
The order in which the operations are performed will determine our final result. Two possibilities are presented below.

Possibility 1

$8 \times \$14 + 4 \times \$17$	
$= \$112 + 4 \times \$17$	Evaluate Multiplication
$= \$116 \times \$17$	Evaluate Addition
$= \$1\ 972$	Evaluate Multiplication

Possibility 2

$8 \times \$14 + 4 \times \$17$	
$= \$112 + 4 \times \$17$	Evaluate Multiplication
$= \$112 + \$68$	Evaluate Multiplication
$= \$180$	Evaluate Addition



Obviously only one answer can be right! Unfortunately for the person involved, **Possibility 2** is the correct answer and the amount earned is \$180 for the 12 hours work.

Mathematical expressions are written to convey specific information, therefore everyone reading them needs to interpret them the same way. For this reason, mathematicians have established a **convention** (an accepted method) that specifies the order in which operations are to be performed.

This **order of operations convention** can be stated as:

When **working from left to right**

Step 1 Evaluate any expressions in brackets* first.
Step 2 Evaluate any powers and roots.
Step 3 Evaluate any multiplications or divisions.
Step 4 Evaluate any additions or subtractions.

* If there are brackets inside another set of brackets, do the inside brackets first.

If you check Possibility 2 above, you will see that it follows the convention for the order of operations, whereas Possibility 1 does not.

This is a very important aspect of mathematics that you must ensure that you understand.

Follow through the examples below carefully.

Example

Evaluate $12 + 2 \times 3$

$12 + 2 \times 3$ Evaluate the multiplication first.
$= 12 + 6$ Finally evaluate the addition.
$= 18$

Let's check this answer on the calculator. Your calculator **automatically** applies the order of operation convention.

On your calculator press 1 2 + 2 x 3 =

The display should read 18

Example

Evaluate $8 - 12 \div (7 - 4)$

$8 - 12 \div (7 - 4)$ Evaluate the brackets first.
$= 8 - 12 \div 3$ Next do the division.
$= 8 - 4$ Finally the subtraction.
$= 4$

To check this on the calculator you will need to find the keys for brackets on your calculator.

Notice that there are opening brackets as well as closing brackets.

For this example press

8 − 1 2 ÷ (7 − 4) =

The display should read 4.

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Example

Evaluate $12 \div {}^{-}3 \times 5 + 6 - (5 - 8)$

$12 \div {}^{-}3 \times 5 + 6 - (5 - 8)$	Evaluate bracket first.
$= 12 \div {}^{-}3 \times 5 + 6 - {}^{-}3$	Rewrite $6 - {}^{-}3$ as $6 + 3$
$= 12 \div {}^{-}3 \times 5 + 6 + 3$	Evaluate multiplication and division left to right.
$= -4 \times 5 + 6 + 3$	
$= -20 + 6 + 3$	Evaluate additions.
$= -14 + 3$	
$= -11$	

Let's check this on your on your calculator.

An example of keystrokes are:

1 2 ÷ (−) 3 × 5 + 6 − (

5 − 8) =

The display should read −11

(Remember the sequence will change if you are using the +/− key.)

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

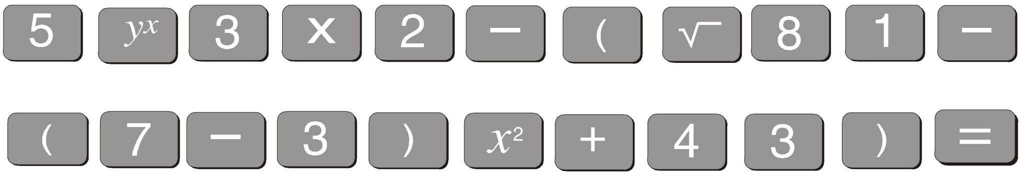
Example

Evaluate $5^3 \times 2 - (\sqrt{81} - (7 - 3)^2 + 43)$

$5^3 \times 2 - (\sqrt{81} - (7 - 3)^2 + 43)$	Evaluate the inside bracket first.
$= 5^3 \times 2 - (\sqrt{81} - 4^2 + 43)$	Evaluate the last bracket. Powers and roots firstly.
$= 5^3 \times 2 - (9 - 16 + 43)$	Now addition and subtraction left to right.
$= 5^3 \times 2 - (-7 + 43)$	
$= 5^3 \times 2 - 36$	Finished brackets. Evaluate powers and roots.
$= 125 \times 2 - 36$	Evaluate multiplication and division.
$= 250 - 36$	Finally addition and subtraction.
$= 214$	

Let's now check this on the calculator.

An example of keystrokes are:



The display should read 214.

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Activity 2.16

1. Evaluate the following **without** a calculator. Check your results on the calculator.

(a)	$7 \times 5 + 4$	(f)	$-3 \div {}^-1 + 9^2 \times {}^-2$
(b)	$10 - 6 \times 7$	(g)	$27 - \sqrt{9} + 21 \div {}^-3$
(c)	$6 + (3 - 9)$	(h)	$4 \times (2 + 5) \div (3 + 1)$
(d)	$-2 - 2 \times {}^-3$	(i)	$(17 - 5^3) \div {}^-3 + 9 \times {}^-5$
(e)	$9 \div 3 \times 7 + 3$	(j)	$(-5 \times {}^-4)^2 - 6 \div (-3 + \sqrt{4})$

2. Evaluate the following **without** a calculator. Estimate your answer before calculating. Check your results on the calculator.

   (a) $765 \div 15 + 822$

   (b) $89 + 21 - 48 \times 23$

   (c) $591 + 37^2 \times \sqrt{49}$

   (d) $4\,763 + 395 \div 5 \times 16$

   (e) $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}) + 61 \times 453$

3. At Andy’s Engineering Works the hourly rate for workers is $14. Overtime is paid at $21 per hour, while working on a public holiday pays $28 per hour.

   (a) Ahmid works as a spray painter. In one busy week Ahmid worked 32 normal hours and 8 hours on Monday which was a public holiday. How much money did Ahmid earn in this week.

   (b) Mary works as an electrician. To catch up on outstanding work Mary agreed to work 8 hours on Show Day, a public holiday. She worked the other four days of the week at 8 hours of normal time and 2 hours of overtime each day. How much money did Mary earn in this week.

4. A passenger coach can carry a maximum of 43 passengers. If the average passenger weighs 54 kg and carries luggage weighing 12 kg what is the usual ‘load’ for the full coach.

5. An express coach from Brisbane to Melbourne stops to load and unload passengers only in Sydney. What was the total amount paid by people using this coach?

Single Fares:

Brisbane – Melbourne	$120
Brisbane – Sydney	$75
Sydney – Melbourne	$68

	Number of Passengers	
City	**Loaded**	**Unloaded**
Brisbane	36	
Sydney	23	14
Melbourne		

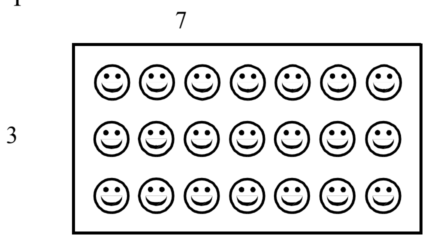
Check out the resource CD and in particular the ‘More than just numbers’ interactive program. You will find this in the ‘Course resources’ section of the CD. When you open the ‘More than just numbers’ program, choose ‘Topics’ and work through the ‘Order of calculations’ topic.

2.4.1 The distributive law

We have been using the order of operation convention to evaluate expressions involving more than one type of operation. Now we will investigate the Distributive Law which uses this convention as its basis.

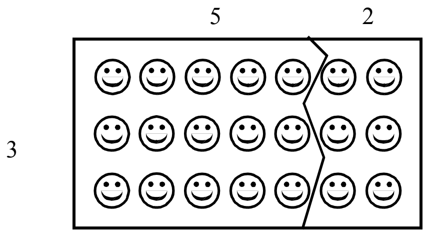
Consider the following situation. In a choir there are twenty-one people arranged in 3 rows of 7 as in Step 1. In Step 2, a rearrangement is being planned. Instead of 3 rows of 7 it is planned to have 2 separate groups (sopranos and altos) – one having 3 rows of 5 and the other with 3 rows of 2. Step 3 shows this rearrangement completed.

Step 1



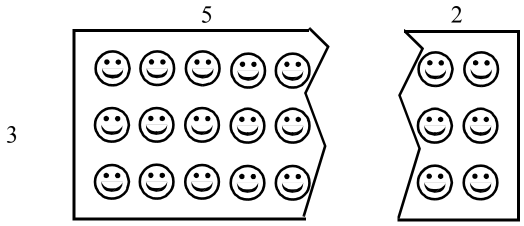
3 × 7

Step 2



3 × (5 + 2)

Step 3



3 × 5 + 3 × 2

Looking at Step 2, we can write

3 × (5 + 2) as 3(5 + 2). The multiplication sign is **understood**.

From Step 3 we can say that:

3(5 + 2)

= 3 × (5 + 2)

= 3 × 5 + 3 × 2 The 3 has been distributed to both the 5 and the 2

= 15 + 6 Applying the order convention.

= 21

Recall that we began with 21 items, so we have distributed the 3 correctly.

We can summarise the distributive law to be:

When terms inside the brackets are added or subtracted, everything inside that bracket is multiplied by whatever is outside the brackets.

Example

Use the distributive law to evaluate 5(9 – 2)

Applying order convention we know that $5(9 - 2) = 5 \times 7 = 35$

Using the distributive law

$5(9 - 2)$
$= 5 \times (9 - 2)$
$= 5 \times 9 - 5 \times 2$ Distribute the 5 to the 9 and the 2
$= 45 - 10$
$= 35$

You could check this on your calculator.

Example

Apply the distributive law to −2(5 + 2)

Using the order convention we know the answer will be $-2 \times 7 = -14$

Using the distributive law

$-2(5 + 2)$
$= -2 \times (5 + 2)$
$= -2 \times 5 + {}^{-}2 \times 2$ Distribute the −2 to the 5 and the 2
$= -10 + {}^{-}4$
$= -14$

Check on your calculator.

Example

Apply the distributive law to −4(6 – 5)

Using the order convention we know the answer will be $-4 \times 1 = -4$

Using the distributive law

$-4(6 - 5)$
$= -4 \times (6 - 5)$
$= -4 \times 6 - {}^{-}4 \times 5$ Distribute the –4 to the 6 and the 5
$= -24 - {}^{-}20$
$= -24 + 20$
$= -4$

Check on your calculator.

A practical use of the distributive law that we will look at in a later module is to think of the day at the variety store when they have 15% off everything.

If you bought a towel, a shirt and a plant you would get 15% off each item.

That is: 15% off towel + 15% off shirt + 15% off plant.

But what the shop does is add up the total cost of the towel, shirt and plant and then deduct the 15% of the total.

That is 15% off (towel + shirt + plant).

We will look at percentages in detail in module 4.

Activity 2.17

1. Apply the distributive law to the following.

   (a) $2(5 + 7)$
   (b) $3(2 + 6)$
   (c) $5(7 - 2)$
   (d) $-2(3 + 5)$
   (e) $-7(5 + 2)$
   (f) $-2(3 - 6)$

2. Apply the distributive law to the following

   (a) $4 \times 8 + 4 \times 6$
   (b) $3 \times 6 - 3 \times 7$
   (c) $-5 \times 8 + {}^{-}5 \times 14$
   (d) $6 \times 48 + 6 \times 57$
   (e) $-17 \times 8 - {}^{-}17 \times 61$

3. Three pizzas are each cut into 6 pieces and another three pizzas are each cut into 8 pieces. How many pieces of pizza are there?

You should now be ready to attempt questions 1, 2, 3, and 4 of Assignment 1A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

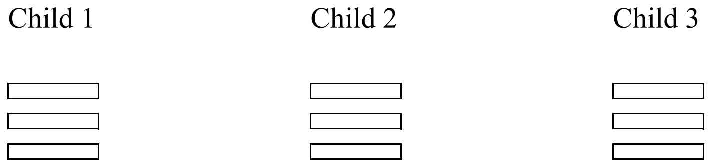
2.5 Fractions

So far in this module we have only looked at whole numbers both positive and negative. There are many real life situations though when whole numbers are not enough. For example consider the situation where a parent might have 10 lollies to be divided amongst 3 children. It is apparent that the 10 lollies cannot be divided up evenly if we are to keep them as wholes. If all the lollies are to be used, each child must also receive a part of a lolly (unless of course the parent eats one and the problem would no longer exist!).

In this section and the next we will look more closely at the ways in which we can represent these parts of the whole.

Consider the problem above.

From the 10 lollies each child will receive 3 whole lollies.



There is one lolly remaining and it must be divided between the three children. Let's look more closely at this lolly (we'll enlarge it).

If we divide the lolly into three parts then each child can receive one of these parts.

Since we have three parts and each child receives one of these we say the child has received one part of three or one third of the lolly.

We can write this as $\frac{1}{3}$. We call this a **fraction** and we say *one third* or *one over three.*

Now for another example.

What part of this figure has been shaded?

Did you say three parts out of 5? Maybe you wrote $\frac{3}{5}$. Either is quite correct.

We will now look at the special names we have for the parts of a fraction.

Consider $\frac{3}{4}$

It is in the form $\frac{\text{numerator}}{\text{denominator}}$

The **denominator** (the number on the bottom of the fraction) tells us how many parts the object is divided into, while the **numerator** (the number on the top of the fraction) tells us how many of these parts we have.

In the case of $\frac{3}{4}$, the denominator is 4 so it is divided into 4 parts and the numerator is 3 so we have 3 of those 4 parts. This is shown below.

We call numbers written in the form $\frac{\text{numerator}}{\text{denominator}}$ **rational numbers.**
We should note here that whole numbers can also be written as fractions. We can write any whole number as being over one.

That is $4 = \frac{4}{1}$, $36 = \frac{36}{1}$ and $-673 = \frac{-673}{1}$

We can see that the rational numbers include the integers that we have already looked at.

Did you know:

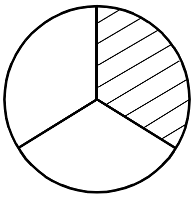
- An iceberg has about $\frac{4}{5}$ of its size under the water. You can see only $\frac{1}{5}$ of the iceberg.
- In a new-born baby, the head is $\frac{1}{4}$ of its total length. The head is $\frac{1}{6}$ of total length at six years and in an adult only $\frac{1}{8}$.
- Gravity on the moon is $\frac{1}{6}$ that on earth. This is what makes people appear to 'float on the moon'.

Check out the resource CD and in particular the 'More than just numbers' interactive program. You will find this in the 'Course resources' section of the CD. When you open the 'More than just numbers' program, choose 'Topics' and work through the 'Fractions' topic.

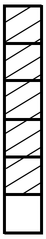
Activity 2.18

1. For each of the following diagrams determine the fraction shaded.

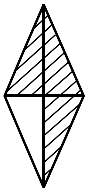
(a)



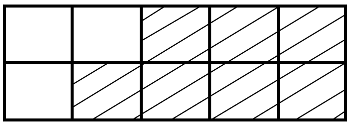
(b)



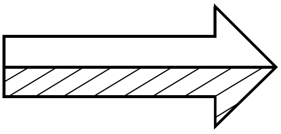
(c)



(d)



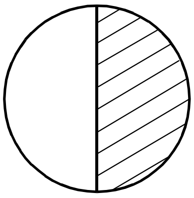
(e)

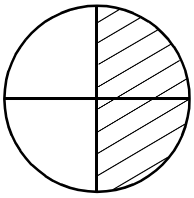


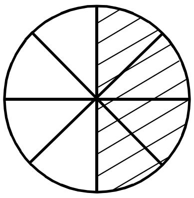
2. Craig ate $\frac{5}{8}$ of the pizza. Draw a diagram to represent the pizza and shade the portion Craig ate.

2.5.1 Equivalent fractions

Consider the following diagrams.







In each case half of the circle has been shaded.

In the first circle this has been represented as 1 part out of 2 or $\frac{1}{2}$.

In the second circle this has been represented as 2 parts out of 4 or $\frac{2}{4}$.

And in the third circle this same amount of shading has been represented as 4 parts out of 8 or $\frac{4}{8}$.

Therefore

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

We say that such fractions are **equivalent fractions**, that is they have different names for the same value. We can write a series of equivalent fractions by multiplying or dividing both the numerator and the denominator by the same number.

These first examples form equivalent fractions by multiplying.

Examples

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Multiplying by $\frac{2}{2}$ is the same as multiplying by 1.

$$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

$$\frac{1}{3} = \frac{1 \times -5}{3 \times -5} = \frac{-5}{-15}$$

$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

Example

Express $\frac{2}{7}$ as an equivalent fraction with a denominator of 21

$$\frac{2}{7} = \frac{?}{21}$$

In this case you would think, ‘what did I multiply 7 by to get 21?’ You should have answered yourself with ‘3’ since 7 × 3 = 21. Those multiplication tables will be very helpful for this section.

Now since we have multiplied the bottom by 3 we must multiply the top by 3 to maintain the balance.

$$\frac{2 \times 3}{7 \times 3} = \frac{6}{21}$$

Therefore $\frac{2}{7} = \frac{6}{21}$

We could read this as $\frac{2}{7}$ equals $\frac{6}{21}$ or we could read this as $\frac{6}{21}$ equals $\frac{2}{7}$. When two parts are equal it doesn’t matter which way we write or read them.

Remember, we have not changed the fraction. Eating $\frac{2}{7}$ of the pizza is the same as eating $\frac{6}{21}$ of it, we just have smaller pieces.

Now to look at some examples that form equivalent fractions by dividing.

The process of dividing the numerator and denominator by the same number is called **simplifying fractions**. We do this until it cannot be done any further. The result is a fraction in simplest form that is equivalent to the original fraction. Recall that we have a special name for a number that divides evenly into another. It is called a **factor**.

Examples

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

We can divide top and bottom by 2 since 2 divides evenly into both 6 and 10.

We say that 2 is a common factor of both 6 and 10.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

We can divide top and bottom by 4 since 4 divides evenly into both 8 and 12.

We call 4 the common factor.

Example

$$\frac{12}{24} = \frac{12 \div 4}{24 \div 4} = \frac{3}{6}$$

We can divide top and bottom by 4 since 4 divides evenly into both 12 and 24.

If you look again at the fraction $\frac{3}{6}$ you will notice that 3 will now divide into top and bottom. The fraction is not in its simplest form.

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

So $\frac{12}{24} = \frac{1}{2}$

It is not always easy to find the largest number that will divide into both the numerator and denominator. As you can see from the example above it doesn't matter if you take more than one step to get to an equivalent fraction in its simplest form.

Here are a few tips for division that may help you to find numbers that will divide into the numerator and denominator.

- A number is divisible by 2 if the number is an **even** number.
- A number is divisible by 3 if the sum of the digits of the number is divisible by 3. For example, 591 is divisible by 3 since 5 + 9 + 1 = 15 which is a number divisible by 3.
- A number is divisible by 4 if the number formed by the last two digits is divisible by 4. For example, 2 732 is divisible by 4 since the last two digits, 32, form a number divisible by 4.
- A number is divisible by 5 if it ends in 0 or 5.
- A number is divisible by 10 if it ends in 0.

We have a shorthand way of writing this process of dividing top and bottom by the same number. We call it cancelling down.

Examples

$\frac{6}{10} = \frac{\cancel{6}^{\,3}}{\cancel{10}_{\,5}} = \frac{3}{5}$ Divide numerator and denominator by 2.

$\frac{-12}{15} = \frac{\cancel{-12}^{\,-4}}{\cancel{15}_{\,5}} = \frac{-4}{5}$ Divide numerator and denominator by 3 because it goes evenly into –12 and 15.

$\frac{-35}{56} = \frac{\cancel{-35}^{\,-5}}{\cancel{56}_{\,8}} = \frac{-5}{8}$ Divide numerator and denominator by 7 because it goes evenly into –35 and 56.

Activity 2.19

1. Write equivalent fractions to each of the following by making the new denominator the number given in brackets.

(a) $\frac{2}{3}$ (9) (d) $\frac{-2}{7}$ (28)

(b) $\frac{4}{5}$ (20) (e) $\frac{7}{12}$ (60)

(c) $\frac{-3}{4}$ (8)

2. Simplify the following fractions, giving your answer in its simplest form.

(a) $\frac{9}{15}$ (d) $\frac{-12}{16}$

(b) $\frac{-3}{18}$ (e) $\frac{30}{40}$

(c) $\frac{8}{10}$ (f) $\frac{2\,000}{6\,000}$

3. The following table shows the different items that have been found around the necks of 75 seals in the wild. This is a very serious problem in the wild and often leads to the death of the seal.

Item	Number
Trawler nets	26
Packaging bands	15
Gillnet	8

Rope	5
Other	21
Total	**75**

(a)What fraction of the seals are strangled by Trawler nets? Remember to write as a fraction you will have to ask 'how many are in the Trawler net category' out of 'how many seals in the total group'.

(b)What fraction of the seals are strangled by Packaging bands? Reduce this fraction to its simplest form.

2.5.2 Mixed numbers and improper fractions

So far we have only looked at fractions where the numerator is **less than** the denominator. For example:

$$\frac{2}{5}, \frac{-3}{8}, \frac{5}{102}$$

We call these types of fractions **proper fractions**. You will notice that a proper fraction will always be less than one.

Consider now the following fractions where the numerator is **greater than** the denominator.

$$\frac{5}{3}, \frac{9}{4}, \frac{300}{250}$$

We call this type of fraction an **improper fraction**.

Let's return to our initial problem of the lollies.

We had 10 lollies and we were to **divide** them amongst 3 children.

That is $10 \div 3$

Fractions are just another way to write division so we could write this expression as

$\frac{10}{3}$ meaning 10 items divided among 3

This fraction is expressed as an improper fraction since the numerator is greater than the denominator.

We also know that each child received 3 whole lollies and $\frac{1}{3}$ of the lolly left over. We say *three and one third* meaning three plus one third.

That is, each child received $3\frac{1}{3}$ lollies. We have expressed the improper fraction $\frac{10}{3}$ as the **mixed number** $3\frac{1}{3}$. We call this a mixed number because it is a mixture of a fraction and a whole number. Other examples of mixed numbers are

$$4\frac{5}{6}, -42\frac{3}{7}, 502\frac{2}{5}$$

Note that for $-42\frac{3}{7}$ the whole fraction is negative, not just the 42. That is $-(42\frac{3}{7})$

Let's look more closely at converting $\frac{10}{3}$ to a mixed number.

$$\frac{10}{3}$$

$$= 3\frac{1}{3}$$

1. How many lots of 3 are there in 10?

   3 with 1 left over as we determined before.
2. Write down the 3 as the whole number part.
3. The remainder 1 becomes the numerator of the fractional part whose denominator is 3.

Check: Is it in its simplest form? Yes!

Let's look at some further examples.

Example

Express $\frac{11}{6}$ as a mixed number

$$\frac{11}{6}$$

$$= 1\frac{5}{6}$$

1. How many lots of 6 are there in 11? 1 with 5 left over.
2. Write down the 1 as the whole number part.
3. The remainder 5 becomes the numerator of the fractional part whose denominator is 6.

Check: Is it in its simplest form? Yes!

Example

Express $\frac{-19}{7}$ as a mixed number

$$\frac{-19}{7}$$

$$= -2\frac{5}{7}$$

1. Write the negative sign at the front of the whole number position.
2. How many lots of 7 are there in 19? 2 with 5 left over.
3. Write down the 2 as the whole number beside the negative sign.
4. The remainder 5 becomes the numerator of the fractional part whose denominator is 7.

Check: Is it in its simplest form? Yes!

Before we move on to some practice in changing improper fractions to mixed numbers, let's look at moving from a mixed number to an improper fraction.

Example

Express $2\frac{3}{4}$ as an improper fraction.

Let's think of $2\frac{3}{4}$ hours and we need to find this in quarter hours.

Now 2 can be written as $\frac{2}{1}$ which is equivalent to $\frac{8}{4}$ or 8 quarter hours.

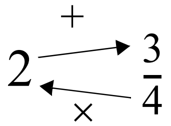
So $2\frac{3}{4}$ hours will be 2 hours plus $\frac{3}{4}$ hour which is 8 quarter hours plus 3 quarter hours, giving us 11 quarter hours.

That is $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$ We will look at this more closely in the next section.

This is the formal method, in practice what we do is multiply the whole number (2) by the denominator of the fractional part (4) and add the numerator (3). This gives the new numerator (11) and the denominator remains the same (4)

Let's look at that in symbols:

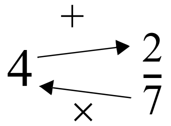
$$2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4}$$



Example

Express $4\frac{2}{7}$ as an improper fraction.

$$4\frac{2}{7} = \frac{4 \times 7 + 2}{7} = \frac{30}{7}$$



Activity 2.20

1. Express these improper fractions as mixed numbers

(a) $\frac{5}{3}$

(b) $\frac{45}{20}$

(c) $\frac{-16}{7}$

(d) $\frac{40}{25}$

(e) $\frac{-35}{14}$

2. Express these mixed numbers as improper fractions

   (a) $3\frac{1}{4}$

   (b) $9\frac{2}{11}$

   (c) $-5\frac{2}{3}$

   (d) $102\frac{5}{12}$

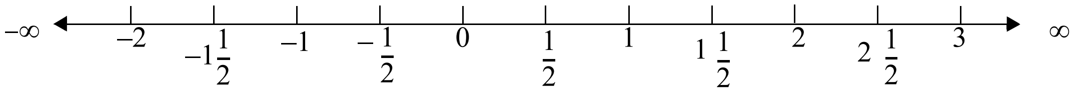
   (e) $-21\frac{2}{5}$

3. If I have $5\frac{2}{3}$ cakes, how many thirds do I have?

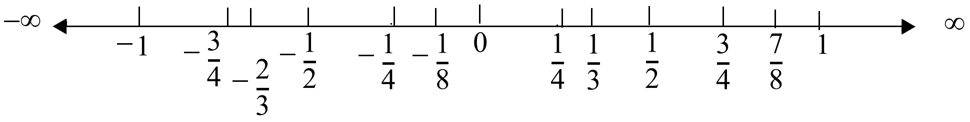
4. Gary is $3\frac{5}{12}$ years old. How old is Gary in months (convert to twelfths)?

Extending the number line

We can now look again at our number line and include the fractions.



Let's look at part of our number line in more detail.



Only a few of the possible fractions have been located on the number line. As there are an infinite number of rationals it would be impossible to locate them all.

We now have on our number line, the integers (positive and negative whole numbers and zero) and the fractions.

We will return again to the number line in a later section.

2.5.3 Calculating with fractions

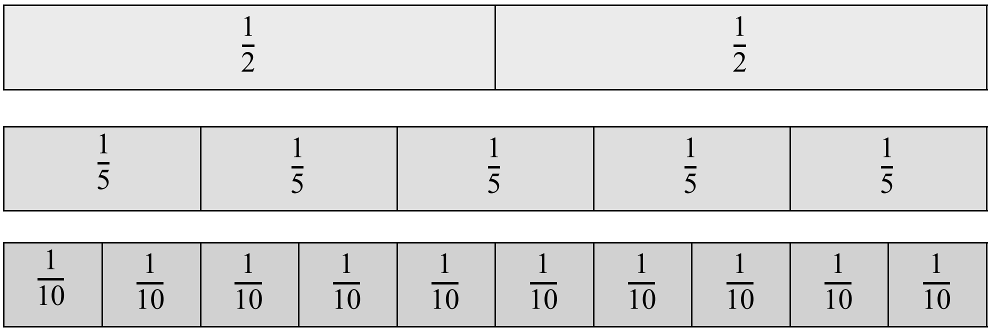
As with whole numbers it is important to be able to perform the four basic operations on fractions without a calculator. We will guide you through the basic operations and show you how to use the calculator. These are essential concepts for all students. For students whose faculty does not permit the use of calculators these skills of manipulation are vital. They will also be used in future work in algebra that you may come across in other levels of mathematics.

Addition and subtraction of fractions

Consider the following:

A wood turner has three pieces of timber of equal length, each a different type of wood. From this timber the wood turner is going to make a decorative panel for a sideboard.

The wood turner divided each piece of wood into even divisions. The first piece of wood into 2 pieces, the second into 5 pieces and the third into 10 pieces.



The wood turner wants to know if the decorative panel will be as long as the original pieces of wood.

Mathematically, the panel consists of the following parts.

$$\frac{1}{10} \quad + \quad \frac{1}{5} \quad + \quad \frac{1}{2} \quad + \quad \frac{1}{5} \quad + \quad \frac{1}{10}$$

That is $\frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

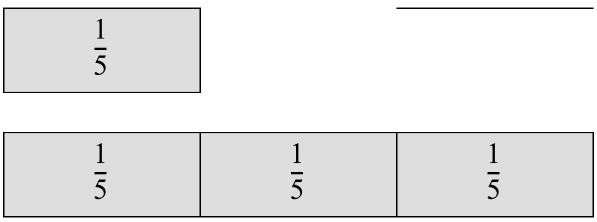
Does this equal the original length, one whole?

Before we can solve this dilemma, we have to look at the fractions differently.

Firstly let's look at a simpler example.

$$\frac{2}{5} + \frac{1}{5}$$

If we take $\frac{2}{5}$



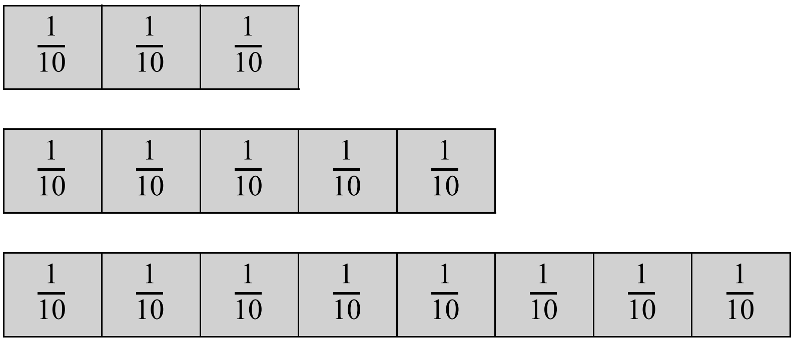
and add $\frac{1}{5}$

we end up with $\frac{3}{5}$

We can write this as $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

Consider the following sum

$$\frac{3}{10} + \frac{5}{10} =$$



We have $\frac{3}{10}$

plus $\frac{5}{10}$

to give $\frac{8}{10}$

That is $\frac{3}{10} + \frac{5}{10} = \frac{8}{10} = \frac{4}{5}$ when written in its simplest form.

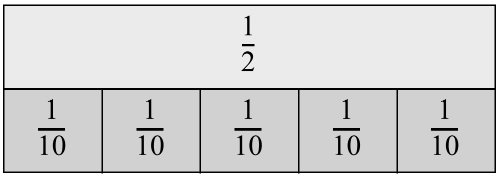
In general, if the denominators are the same we need only add or subtract the numerators to find the answer. We should then reduce the fraction to its simplest form if necessary.

Much of the time though, the fractions we need to add have different denominators.

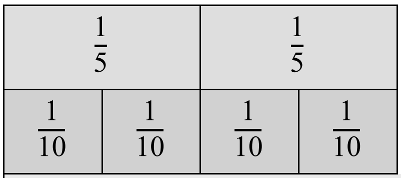
Consider the following example

$$\frac{1}{2} + \frac{2}{5} =$$

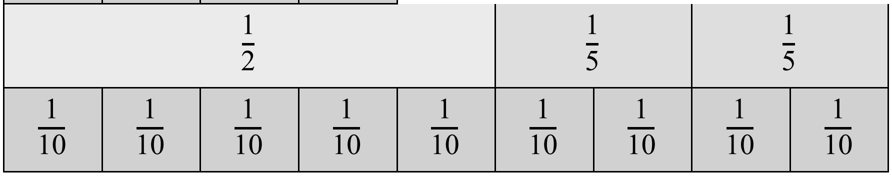
We cannot work out a single answer from the above diagram. Consider changing both to tenths.



$\frac{1}{2}$ can be expressed as $\frac{5}{10}$



$\frac{2}{5}$ can be expressed as $\frac{4}{10}$



giving a total of $\frac{9}{10}$

We can write this as $\frac{1}{2} + \frac{2}{5}$

$$= \frac{5}{10} + \frac{4}{10}$$

$$= \frac{9}{10}$$

We have changed each of the fractions into an equivalent fraction with the new denominator 10.

That is $\frac{1 \times 5}{2 \times 5} + \frac{2 \times 2}{5 \times 2}$

$$= \frac{5}{10} + \frac{4}{10}$$

$$= \frac{9}{10}$$

We call 10 the **lowest common denominator** (LCD) of $\frac{1}{2}$ and $\frac{2}{5}$. That is, 10 is the smallest number divisible by both denominators, 2 and 5.

Consider the following examples.

For $\frac{2}{3}$ and $\frac{1}{2}$ the lowest common denominator is 6, since 6 is the smallest number divisible by both 3 and 2.

For $\frac{3}{4}$ and $\frac{5}{8}$ the lowest common denominator is 8, since 8 is the smallest number divisible by both 4 and 8.

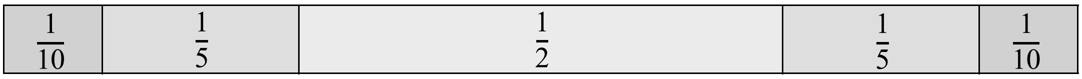
For $\frac{7}{12}$ and $\frac{3}{8}$ the lowest common denominator is 24, since 24 is the smallest number divisible by both 12 and 8.

It is often difficult to find the lowest common denominator for any two numbers. It is really quite acceptable to use any common denominator. If you do not find the lowest of the common denominators it will simply mean that you will have to simplify your final answer.

So in general, to add or subtract fractions with different denominators, we change each fraction into an equivalent fraction with the **lowest common denominator** (LCD) and then add or subtract the numerators and simplify if necessary.

Consider the following examples.

Let's return to our wood turner.



That is, $\frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

If we expressed each of these as tenths we could add the parts:

$$\frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{5}{10} + \frac{2}{10} + \frac{1}{10} = \frac{11}{10}$$

In fact the decorative panel is a bit longer than the original pieces of wood. One length of wood is $\frac{10}{10}$ and the panel is $\frac{11}{10}$.

Example

Evaluate $\frac{1}{2} + \frac{1}{4}$

$\frac{1}{2} + \frac{1}{4}$ The LCD of $\frac{1}{2}$ and $\frac{1}{4}$ is 4

$$= \frac{1 \times 2}{2 \times 2} + \frac{1}{4}$$

$= \frac{2}{4} + \frac{1}{4}$ $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions

$= \frac{3}{4}$

It is possible to enter fractions on the calculator and hence to work with them as we have done in past sections.

Find the $a\,b/c$ key on your calculator

For the $a\,b/c$ key the ‘a’ represents the whole number part of a mixed number and the ‘b/c’ represents the fractional part.

To evaluate $\frac{1}{2} + \frac{1}{4}$ press 1 $a\,b/c$ 2 + 1 $a\,b/c$ 4 =

The display should read 3⌋4 which you would interpret as meaning $\frac{3}{4}$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Example

Evaluate $\frac{3}{4} - \frac{1}{6}$

$\frac{3}{4} - \frac{1}{6}$ The LCD of $\frac{3}{4}$ and $\frac{1}{6}$ is 12

$= \frac{3 \times 3}{4 \times 3} - \frac{1 \times 2}{6 \times 2}$

$= \frac{9}{12} - \frac{2}{12}$

$= \frac{7}{12}$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

3 ab/c 4 − 1 ab/c 6 = The display should read 7⌟ 12

At other times it is necessary to add or subtract mixed numbers. To do this, convert the mixed numbers to improper fractions and then continue as before.

Example

Evaluate $3\frac{2}{5} + 2\frac{3}{8}$

$3\frac{2}{5} + 2\frac{3}{8}$

$= \frac{17}{5} + \frac{19}{8}$ Convert to improper fractions

$= \frac{17 \times 8}{5 \times 8} + \frac{19 \times 5}{8 \times 5}$ LCD is 40

$= \frac{136}{40} + \frac{95}{40}$

$= \frac{231}{40}$ Your calculator can help convert to a mixed number if you press 231 a b/c 40 = your calculator gives you the mixed number.

$= 5\frac{31}{40}$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

3 ab/c 2 ab/c 5 + 2 ab/c 3 ab/c 8 =

The display should read 5⌟31⌟40

Example

Evaluate $-5\frac{1}{4} + 3\frac{5}{6}$

$-5\frac{1}{4} + 3\frac{5}{6}$

$= -\frac{21}{4} + \frac{23}{6}$ Convert to improper fractions.

$$= \frac{-21 \times 6}{4 \times 6} + \frac{23 \times 4}{6 \times 4}$$

Although the LCD is 12, we will use 24 to show that it just means an extra cancelling at the end.

$$= \frac{-126}{24} + \frac{92}{24}$$

$-126 + 92 = -34$

$$= \frac{-34}{24}$$

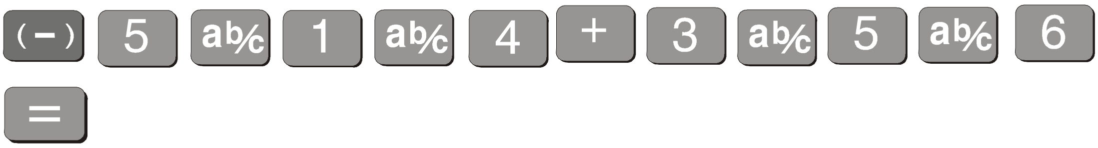
Not taking the LCD means an extra step here. Not really a problem is it?

$$= -\frac{17}{12}$$

$$= -1\frac{5}{12}$$

Now try this on your calculator.

Write down the keystrokes you have used if they are different.



The display should read – 1⌟5 ⌟12

Activity 2.21

1. Evaluate the following without a calculator. Check your answers with a calculator.

(a) $\frac{3}{7} + \frac{1}{2}$

(b) $\frac{2}{3} - \frac{3}{4}$

(c) $\frac{13}{15} + \frac{2}{5}$

(d) $-\frac{5}{6} + \frac{9}{10}$

(e) $\frac{5}{9} - \frac{5}{6} + \frac{1}{2}$

(f) $3\frac{1}{4} + 2\frac{1}{2}$

(g) $5\frac{3}{10} - 4\frac{2}{5}$

(h) $-3\frac{7}{9} + 2\frac{1}{6}$

(i) $17\frac{1}{6} + 25\frac{1}{4}$

(j) $2\frac{5}{7} - 3\frac{2}{9}$

2. If Jason ate $\frac{3}{8}$ of a pizza and Jenny ate $\frac{2}{5}$, who ate the most pizza? (Hint: convert to a common denominator)

3. A patient takes $\frac{1}{3}$ of his medication in the first hour and $\frac{2}{5}$ in the following hour. How much of the medication remains?

4. Sean has worked out a budget that he is keen to follow. He has decided on the following distribution of income:

   savings $\frac{1}{20}$

   rent $\frac{1}{4}$

   food $\frac{1}{6}$

   electricity/car/other bills $\frac{2}{5}$

   entertainment ?

   What fraction of Sean's income will be left for entertainment? Hint: your LCD must apply to all denominators.

5. Four trucks tipped the following loads of clean fill onto a block of land. What was the total amount of fill dumped if the loads were $2\frac{3}{4}$ tonnes, $6\frac{2}{5}$ tonnes, $4\frac{3}{8}$ tonnes and $8\frac{1}{2}$ tonnes?

6. An electrician buys plastic tubing in 6 metre lengths. From one particular piece the electrician has cut pieces of lengths $1\frac{3}{4}$ metres, $2\frac{1}{2}$ metres and $\frac{3}{8}$ metre. How much of the piece of tubing is left?

Multiplication of fractions

Recall from the section on multiplying whole numbers, that multiplication is a shorthand way of writing a repeated addition.

For example $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

could be written as 3 lots of $\frac{1}{5}$ or $3 \times \frac{1}{5}$

Now we know that $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

so $3 \times \frac{1}{5}$ must also equal $\frac{3}{5}$.

To get this we have: $3 \times \frac{1}{5}$

$= \frac{3}{1} \times \frac{1}{5}$ Convert 3 to an improper fraction.

$$= \frac{3 \times 1}{1 \times 5}$$

Multiply numerators.
Multiply denominators.

$$= \frac{3}{5}$$

Let's look at some further examples.

Example

Evaluate $\frac{3}{4} \times \frac{5}{8}$

$$\frac{3}{4} \times \frac{5}{8}$$

$$= \frac{3 \times 5}{4 \times 8}$$

Multiply numerators.
Multiply denominators.

$$= \frac{15}{32}$$

Check this on your calculator.

Example

Evaluate $\frac{7}{8} \times \frac{16}{21}$

$$\frac{7}{8} \times \frac{16}{21}$$

$$= \frac{\overset{1}{\cancel{7}} \times \cancel{16}^{\,2}}{\underset{1}{\cancel{8}} \times \cancel{21}_{\,3}}$$

Cancel common factors. (Divide 16 and 8 by 8, and 7 and 21 by 7).

$$= \frac{1 \times 2}{1 \times 3}$$

$$= \frac{2}{3}$$

Check on your calculator.

Example

Evaluate $-3\frac{1}{2} \times 6\frac{2}{5}$

$$-3\frac{1}{2} \times 6\frac{2}{5}$$

$$= -\frac{7}{2} \times \frac{32}{5}$$

Convert to improper fractions.

$$= \frac{-7 \times \cancel{32}^{\,16}}{\underset{1}{\cancel{2}} \times 5}$$

Cancel common factors. (Divide 32 and 2 by 2).

$$= \frac{-7 \times 16}{1 \times 5}$$

$= -\dfrac{112}{5}$

Did you remember that a negative number multiplied by a positive number gives a negative answer?

$= -22\dfrac{2}{5}$

Example

In a library of 25 000 books $\frac{3}{5}$ were found to be non-fiction. How many books were non-fiction?

$$\text{Non-fiction books} = \frac{3}{5} \times 25\,000$$

$$= \frac{3 \times \cancel{25\,000}^{\,5\,000}}{{}_{1}\cancel{5} \times 1}$$

Cancel common factors.

$$= 3 \times 5\,000$$

$$= 15\,000$$

Therefore there are 15 000 non-fiction books in the library.

Activity

Activity 2.22

1. Evaluate the following without a calculator. Check your answers with a calculator.

   (a) $\dfrac{5}{9} \times \dfrac{3}{20}$

   (b) $\dfrac{3}{8} \times \dfrac{7}{9}$

   (c) $-7\dfrac{3}{5} \times 4\dfrac{1}{2}$

   (d) $-3\dfrac{3}{4} \times -7\dfrac{1}{20}$

   (e) $\dfrac{10}{15} \times -\dfrac{4}{9}$

2. Find

   (a) $\frac{3}{4}$ of 256 metres

   (b) $\frac{4}{9}$ of 594 grams

   (c) $\frac{3}{10}$ of 45 000 hectares

3. Chung decided that he would rest after travelling $\frac{3}{5}$ of his 450 kilometre journey. After how many kilometres did Chung rest?

4. The Smith's swimming pool is in need of repair. The pool maintenance company told the Smith's they would need to reduce the amount of water in the pool by $\frac{1}{3}$. Luckily the Smiths know that their pool holds 51 000 litres of water. How much water must the Smiths drain from their pool?

5. A family holiday cost $1 850. Accommodation took $\frac{3}{5}$ of this, while $\frac{3}{10}$ went on travelling. The remainder was spent on entertainment.

   (a) How much was spent on accommodation?

   (b) How much was spent on travelling?

   (c) What fraction of the holiday money was spent on entertainment?

   (d) How much money was spent on entertainment?

   (e) How could you check that your calculation in (d) was correct?

6. Chris was working on some maths questions and incorrectly did the following calculation.

$$8 \times \frac{3}{4} = \frac{8 \times 3}{8 \times 4} = \frac{24}{32}$$

   (a) What did Chris do wrong?

   (b) How would you explain the error to Chris?

7. The following two statements come from Robert Hughes' book *The Fatal Shore,* (*Collins* Harvill, Great Britain, 1987) about the history of Australia.

   (a) *Of the 137 rebels eventually charged with mutiny, half were lifers and another third had sentences of fourteen years.*

   How many of these Norfolk Island rebels were 'lifers' and how many had sentences of fourteen years.

   (b) ***The First Fleet carried enough food to keep its passengers alive for two years in Australia. The rations issued to sailors, marines and officers each week were:***

Beef4 poundHardtack*7 pound
Pork2 poundCheese12 ounces
Dried peas2 pintsButter6 ounces
Oatmeal3 pintsVinegar $\frac{1}{2}$ ***pint***

The male convicts got one third less, while female convicts got two thirds of the male ration, or slightly less than half the naval standard. On paper, this was not a bad allowance. In practice it meant scurvy, and the meat was mostly bone and gristle.
*Hardtack was a type of hard biscuit.

(i) Calculate the weekly ration of each item for the male convicts.

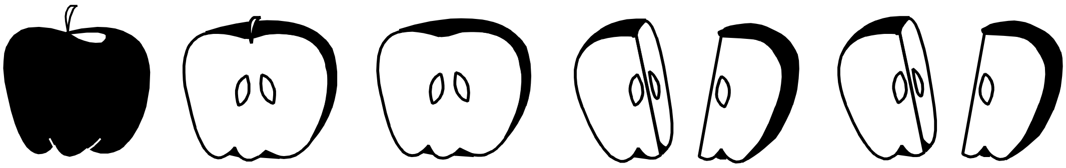
(ii) Calculate the weekly ration of each item for the female convicts.

(iii) Show that the female ration is ‘slightly less than half the naval ration’.

Division of fractions

When we dealt with positive whole numbers, dividing gave us a smaller number. For example, $32 \div 4 = 8$, 8 is a smaller number than the original 32. What happens when we divide by a fraction?

Let’s consider the following apple, cut in half, then into quarters.



How many quarters are there in one whole apple? There are four quarters in one whole apple.

$$1 \div \frac{1}{4} = 4$$

We end up with a larger number than we started with. Very interesting!!

Let’s look at how we can divide without the diagram.

Consider the following **reciprocals.**

The **reciprocal** of $\frac{2}{3}$ is $\frac{3}{2}$

The **reciprocal** of $\frac{14}{9}$ is $\frac{9}{14}$

The **reciprocal** of $\frac{3}{78}$ is $\frac{78}{3}$

Can you describe in your own words what is meant by the term reciprocal?

……………………………………………………………………………………………………

You should have said something like the reciprocal is the fraction turned upside down.

Let’s multiply a fraction by its reciprocal.

$$\frac{2}{3} \times \frac{3}{2}$$

$$= \frac{{}^{1}\cancel{2} \times \cancel{3}^{1}}{{}_{1}\cancel{3} \times \cancel{2}_{1}}$$

$$= \frac{1 \times 1}{1 \times 1}$$

$$= 1$$

Here’s another

$$-\frac{2}{9} \times -\frac{9}{2}$$

$$\frac{{}^{-1}\cancel{-2} \times \cancel{-9}^{-1}}{{}_{1}\cancel{9} \times \cancel{2}_{1}}$$

$$= \frac{-1 \times -1}{1 \times 1}$$

$$= 1$$

In fact, if we multiply any number by its reciprocal, the result is always 1.

However, we also know that anything divided by itself is 1

$$7 \div 7 = 1$$

$$-5 \div -5 = 1$$

$$\frac{2}{3} \div \frac{2}{3} = 1$$

$$-\frac{2}{9} \div -\frac{2}{9} = 1$$

Now since $\frac{2}{3} \div \frac{2}{3} = 1$

and $\frac{2}{3} \times \frac{3}{2} = 1$

we can say that $\frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2}$ because they both give the answer 1.

Similarly $-\frac{2}{9} \div -\frac{2}{9} = 1$

and $-\frac{2}{9} \times -\frac{9}{2} = 1$

therefore we can say that $-\frac{2}{9} \div -\frac{2}{9} = -\frac{2}{9} \times -\frac{9}{2}$

Let's look back to our apple cut into quarters.

$1 \div \frac{1}{4} = 4$ We know there were four quarters in the apple.

so $\frac{1}{1} \times \frac{4}{1} = 4$ The reciprocal method gives us the same result.

In general we can see that dividing by a fraction is the same as multiplying by its reciprocal.

Example

Evaluate $\frac{3}{4} \div \frac{2}{3}$

$\frac{3}{4} \div \frac{2}{3}$

$= \frac{3}{4} \times \frac{3}{2}$ Find the reciprocal of $\frac{2}{3}$ and multiply instead of divide.

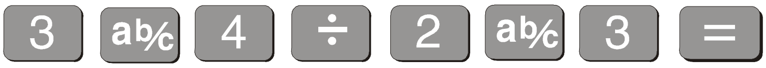
$= \frac{3 \times 3}{4 \times 2}$

$= \frac{9}{8}$

$= 1\frac{1}{8}$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.



Example

Evaluate $-1\frac{2}{3} \div -\frac{25}{27}$

$$-1\frac{2}{3} \div \frac{-25}{27}$$

$$= -\frac{5}{3} \div \frac{-25}{27}$$

Convert the mixed number to an improper fraction.

$$= -\frac{5}{3} \times \frac{-27}{25}$$

Multiply by the reciprocal.

$$= \frac{\overset{-1}{\cancel{-5}} \times \overset{-9}{\cancel{-27}}}{\underset{1}{\cancel{3}} \times \underset{5}{\cancel{25}}}$$

Cancel where appropriate.

$$= \frac{-1 \times -9}{1 \times 5}$$

$$= \frac{9}{5}$$

Did you remember that a negative number multiplied by a negative number gives a positive answer?

$$= 1\frac{4}{5}$$

☑ Now try this on your calculator.

Write down the keystrokes you have used if they are different.

(−) 1 a b/c 2 a b/c 3 ÷ (−) 2 5 a b/c 2 7 =

Activity 2.23

1. Evaluate the following without a calculator. Check your answers on the calculator.

(a) $\frac{3}{4} \div \frac{9}{20}$

(b) $-\frac{21}{25} \div \frac{35}{30}$

(c) $4\frac{2}{5} \div 2\frac{7}{10}$

(d) $7\frac{5}{8} \div -3\frac{1}{2}$

(e) $-2\frac{1}{4} \div -\frac{3}{8}$

2. A 30 metre length of rope is to be divided into $1\frac{1}{5}$ metre lengths. How many pieces can be cut from this piece of rope?

3. A plant grows $1\frac{2}{3}$ centimetres per month. Provided that growing conditions remain the same, how long would it take for the plant to reach $28\frac{3}{4}$ centimetres?

4. Part of the nutritional information on the side of a box containing 750 gram of cereal gives the following:

Protein	90 grams
Fat	21 grams
Carbohydrates	480 grams
Niacin	62 micrograms

(a) If the serving size is 30 grams, what fraction is this of the total weight of cereal in the box?

(b) Using your answer from part (a) find the amount of protein, fat, carbohydrate and Niacin present in one serve of this cereal.

(c) If the recommended daily allowance of Niacin in Australia is 10 micrograms, how many serves of this cereal would I need to have to get all Niacin requirements for the day?

Recall that we used the order of operation convention when dealing with whole numbers. It holds true for all types of numbers including fractions. Refresh your memory on order of operation if you need to before attempting the following activity.

Recall:
Module 2
Section 2.4

Activity 2.24

1. Evaluate the following without a calculator. Check your answers using your calculator.

(a) $\frac{2}{3} \times \left(\frac{1}{2} + \frac{1}{3}\right)$

(b) $1\frac{1}{2} \div \left(\frac{3}{7} - \frac{1}{14}\right)$

(c) $\frac{3}{5} - 2\frac{2}{5} \times \frac{19}{4}$

(d) $\frac{3}{4} \div \frac{6}{5} + \frac{6}{8} \times 3\frac{2}{3}$

(e) $\left(\frac{3}{5}\right)^2 - \left(\frac{4}{5} + \frac{1}{2}\right)$

2. In a survey of 750 teenagers it was found that $\frac{2}{5}$ were heavy smokers, $\frac{1}{3}$ smoked occasionally, and the rest were non-smokers. How many were non-smokers?

3. On a recent trip to the butcher the Brown family made the following purchases.

$3\frac{1}{2}$ kilograms of sausages at $4 per kilogram

4 kilograms of mince at $5 per kilogram

Leg of lamb at $12

$1\frac{3}{4}$ kilograms of pork chops at $8 per kilogram.

How much change did the Browns receive from a $100 note?

2.6 Decimals

Have you ever looked at your bank account (or maybe somebody else's) and seen numbers such as $2 304.27 (don't we wish!!)? This means 2 thousand, 3 hundred and 4 dollars and 27 cents. Why are the cents written $0.27?

In this section we are going to look at these decimal numbers. In fact decimals are just another way of writing fractions, some would say a more convenient way when it comes to calculating.

Decimals are special fractions where the denominators are powers of 10 (e.g. $10 = 10^1$, $100 = 10^2$, $1\,000 = 10^3$).

For example $\frac{2}{10}$ and $\frac{45}{100}$

You will recall from earlier work that a fraction is just another way of writing a division.

So $\frac{2}{10}$ could be written $2 \div 10$. When you do this division on the calculator you should get 0.2

That is $\frac{2}{10} = 2 \div 10 = 0.2$

And when you do the same thing to $\frac{45}{100}$ you will get

$$\frac{45}{100} = 45 \div 100 = 0.45$$

We call this type of number a **decimal** or sometimes a **decimal fraction**.

Let's look back to the place value table from the beginning of this module. We can now extend this table by putting in a **decimal point** and extending our place values to the right.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units	.	tenths	hundredths	thousandths	ten thousandths

Consider the number 356.782

We say this number as ***three hundred and fifty six point seven eight two***. After the decimal point we say the name of the digit as we come to it.

The 7 represents 7 tenths, the 8 represents 8 hundredths and the 2 represents 2 thousandths. (Some of these words are hard to get your tongue around!)

Activity 2.25

1. Write these numbers in the following place value table.
    (a) five hundred and twenty four point one nine
    (b) two thousand point zero zero four
    (c) negative seven hundred and ninety five thousand
    (d) negative sixty six thousand and fifty five point three two
    (e) zero point three zero six

	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units	.	tenths	hundredths	thousandths	ten thousandths
(a)												
(b)												
(c)												
(d)												
(e)												

2. What is the place value of each of the numbers underlined below?

   (a) 79$\underline{5}$.2$\underline{4}$
   (b) $\underline{2}$ 004.$\underline{6}$
   (c) $\underline{8}$2.30$\underline{5}$
   (d) −$\underline{6}$ 953 000.0$\underline{1}$
   (e) −$\underline{1}$2 567.005 $\underline{7}$

Before we do any calculations with decimals it is important that we know how to estimate with them.

We use much the same method as we used with whole numbers. The only difference is that when the round-off position is after the decimal point, we do not fill any spaces with zeros.

Example

Round 3.648 to the nearest tenth.

As before, look at the digit to the right of the tenth position, in this case the 4. It is less than 5 so we leave the digit in the tenths position as it is.

3.648 ≈ 3.6 rounded to the nearest tenth.

Example

Round 3 476.638 to two decimal places.

To end up with two decimal places we are rounding to the nearest hundredth. Look at the digit to the right of the hundredth place, in this case the 8. It is greater than or equal to 5 so we take the round-off place up by one.

3 476.638 ≈ 3 476.64 rounded to two decimal places.

Example

Round 0.003 76 to the leading digit.

Recall that the leading digit is the first non-zero digit.

In this example that would be the 3. Following the normal procedures for rounding we get:

0.003 76 ≈ 0.004 rounded to the leading digit.

Activity 2.26

1. Round these numbers to the place value indicated.
   - (a) 576.205 to the nearest hundredth
   - (b) 75 201.3 to the nearest unit
   - (c) –0.008 to the nearest hundredth
   - (d) 67.345 67 to one decimal place
   - (e) –6 399.998 to two decimal places
2. Round the following numbers to the leading digit.
   - (a) 2 576.205
   - (b) 201.3
   - (c) –0.028
   - (d) 97.345 67
   - (e) –6 009

2.6.1 Addition and subtraction of decimals

The same rules for addition and subtraction of whole numbers that we have previously looked at will apply to decimals. Recall that our steps were to:

- estimate by rounding to the leading digit,
- calculate by lining up the numbers according to place value and finally to
- check for the reasonableness of the answer.

We will look at only one example here and leave you to follow the same steps for the questions in the activity following.

Example

Evaluate 76.4 + 105.92 + 3.456 + 2.1

Estimate:

$$\begin{aligned} 76.4 &\approx 80 \\ 105.92 &\approx 100 \\ 3.456 &\approx 3 \\ 2.1 &\approx 2 \\ 80 + 100 + 3 + 2 &= 185 \end{aligned}$$

Calculate:

$$\begin{array}{r} 76.4 \\ 105.92 \\ 3.456 \\ +\quad 2.1 \\ \hline 187.876 \end{array}$$

1. Line up according to place value.
2. Add as for whole numbers.
3. Insert the decimal point in the correct position.

Check: The answer is close to our estimate and so looks reasonable.
(Now check on the calculator.)

Activity

Activity 2.27

1. Evaluate the following. Estimate results before commencing.

   (a) 9.56 + 2.589 + 13.81

   (b) 15.67 – 2.5

   (c) 148 + 0.003 + 2.6079

   (d) 3.68 – 89.2

   (e) –8 974.2 + 356.47

2. Three teenagers held a sponsored swim for Famine Relief. They raised \$18.57, \$24.32 and \$9.84. How much did they raise?

3. Poison for the garden is purchased in a 2 litre container. If 0.15 litres has been used, how much poison remains in the container? Hint: write 2 litres as 2.00 litres to assist with the subtraction. This does not change the number.

4. The weight loss table for a patient is recorded over a thirty day period at 10 day intervals.

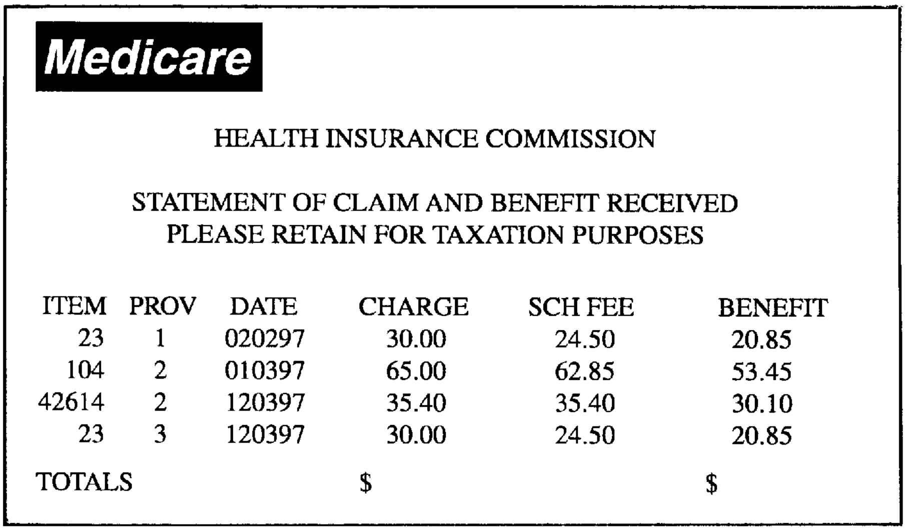
	Weight (kilograms)	**Weight loss**
Start	88.904	0
10th day	87.772	
20th day	86.592	
30th day	84.036	

   (a) Calculate this patient’s weight loss after each recording.

   (b) What is the total weight loss for this patient?

   (c) Round this answer to the nearest tenth of a kilogram.

5. The Youth Club spent \$100.00 preparing for their Saturday disco. Income from the disco was \$64 from ticket sales, \$17.40 from raffle ticket sales and \$16.77 from a refreshment stall at the disco. How much did the youth club have to pay to cover costs?
6. Following is part of a statement received from Medicare after making a claim.



Medicare

HEALTH INSURANCE COMMISSION

STATEMENT OF CLAIM AND BENEFIT RECEIVED
PLEASE RETAIN FOR TAXATION PURPOSES

ITEM	PROV	DATE	CHARGE	SCH FEE	BENEFIT
23	1	020297	30.00	24.50	20.85
104	2	010397	65.00	62.85	53.45
42614	2	120397	35.40	35.40	30.10
23	3	120397	30.00	24.50	20.85
TOTALS			\$		\$

(a) Find the totals for the CHARGE and the BENEFIT columns.

(b) Charge represents the amount of money that the patient paid, while the Benefit is the amount of money that Medicare refunded to the patient. How much of the total bill was left for the patient to pay after the Medicare benefit was allowed?

(c) Many doctors charge more than the government's scheduled fee (Sch Fee). How much more than the scheduled fee has this patient been charged for each item?

2.6.2 Multiplication of decimals

Before we look at some more involved multiplications, let's look at some easy ones.

Multiplying by powers of ten.

Find the answer to the following products on your calculator.

$2.46 \times 10 =$ Note that 10 could be written 10^1
$25.638 \times 10 =$
$5\,694.3 \times 10 =$
$0.008\,94 \times 10 =$
$43 \times 10 =$

Can you see a pattern? Complete the following sentence.

Multiplying by 10 moves the decimal point ....

Try these

2.46 × 100 =   Note that 100 could be written 10^2
25.638 × 100 =
5 694.3 × 100 =
0.008 94 × 100 =
43 × 100 =

Complete the following sentence.

Multiplying by 100 moves the decimal point ....

Finally try these.

2.46 × 100 000 =   Note that 100 000 could be written 10^5
25.638 × 100 000 =
5 694.3 × 100 000 =
0.008 94 × 100 000 =
43 × 100 000 =

Complete the following sentence.

Multiplying by 100 000 moves the decimal point ....

We can generalize these rules for multiplying a number by a power of ten.

> When **multiplying** by a power of 10, move the decimal point to the right by the number of zeros in the power of 10.

In general to multiply decimals:

- Estimate the answer as before.
- Ignoring the decimal points, multiply the numbers as if they were whole numbers.
- Count the number of decimal places in the question.
- Position the decimal point to give the same number of decimal places as in the question.
- Check your answer against your estimate.

Example

Evaluate 125.6 × 79.04

Estimate:

$$\begin{aligned} 125.6 &\approx 100 \\ 79.04 &\approx 80 \\ 100 \times 80 &= 8\,000 \end{aligned}$$

Calculate:

$$\begin{array}{r}
125.6\\
\times\,\underline{79.04}\\
{\scriptstyle 2\;2}\\
4804\\
{\scriptstyle 1\;4\;5}\\
985400\\
{\scriptstyle 1\;3\;4}\\
7452000\\
\underline{{\scriptstyle 1\;2\;1\;1}}\\
9927.424
\end{array}$$

We do not need to write in the row of zeros when multiplying be zero. We can move on to multiply by 900 by writing down two zeros and multiplying by 9.

There are three decimal places in the question so place the decimal point to give three decimal places in the answer.

Check: Yes, this answer is close to the estimate so looks reasonable.
(Check on your calculator.)

Activity 2.28

1. Evaluate the following. Estimate results before commencing.

   (a) 52.6×3.95

   (b) -2.05×20

   (c) $0.56 \times {}^{-}1.23$

   (d) 145×1.58

   (e) $-0.025 \times - 3.6$

2. Olga makes monthly repayments of $37.45 on a loan for a stereo. She takes 2 years to pay for the stereo.

   (a) Estimate the total cost.

   (b) Calculate the exact total cost.

3. If 1.5 millilitres of an experimental drug is given to each of 12 patients each day, how many millilitres of the drug will be used in 20 days?

4. A book has 367 pages. If each page is 0.1 millimetres thick and the covers are each 0.2 millimetres thick, what is the total thickness of the book?

5. Irving makes $14.68 an hour for the first 40 hours in a week and time and a half for each hour over 40 that he works in one week.

   (a) Estimate Irving's salary for a 40 hour week.

   (b) Calculate his exact salary for a 40 hour week.

   (c) Calculate his salary in a week in which he works 50 hours.

6. Following is the pricing schedule for Australia Post’s Express Post envelopes.

Quantity purchased	Single item	Packs of 10		
		1–4 packs	5–9 packs	10 or more packs
Discount	none	5%	10%	15%
Price	per item	per pack	per pack	per pack
Envelopes				
C5	$3.00	$28.50	$27.00	$25.50
B4	$4.00	$38.00	$36.00	$34.00
Satchels				
500 g	$5.00	$47.50	$45.00	$42.50
3 kg	$7.70	$73.15	$69.30	$65.45
			Packs of 50	
Window faced envelopes	Only sold in packs of 50	1 pack	2 or more packs	Box of 500 (per box)
DL	na	$135.00	$127.50	$1 275.00

(a) I wish to purchase twenty 500 g satchels.

(i) What would the cost be if I purchased them individually?
(ii) What would the cost be if I purchased packs of 10?
(iii) What is the saving in purchasing in bulk?

(b) Find the cheapest way of purchasing the following order:

Eight 3 kg satchels
One hundred C5 envelopes
One hundred DL envelopes.

7. The Toowoomba City Council charges consumers for water according to the amount they use.

1st Tier	0 to 324 kilolitres	$0.35 per kilolitre
2nd Tier	above 324 kilolitres	$1.00 per kilolitre

The council officer writes down the following readings for the meter at the Kennedy household.

Previous reading	3 619 kilolitres
Present reading	3 981 kilolitres

Calculate the water bill for the Kennedy residence.

8. Following are the taxation rates payable for residents for the 1996/7 financial year.

Taxable income	Tax payable
\$0 – \$5 400	Nil
\$5 401 – \$20 700	20c for each \$1 in excess of \$5 400
\$20 701 – \$38 000	\$3 060 + 34c for each \$1 in excess of \$20 700
\$38 001 – \$50 000	\$8 942 + 43c for each \$1 in excess of \$38 000
Over \$50 000	\$14 102 + 47c for each \$1 in excess of \$50 000

Calculate the tax payable for people earning the following taxable incomes.

(a) \$4 356
(b) \$34 780
(c) \$48 940
(d) \$56 200

2.6.3 Division of decimals

Before we look at some more involved divisions, let's look at dividing by powers of ten.

Find the answer to the following divisions on your calculator.

$2.46 \div 10 =$ Note that 10 could be written 10^1
$25.638 \div 10 =$
$5\,694.3 \div 10 =$
$0.008\,94 \div 10 =$
$43 \div 10 =$

Can you see a pattern? Complete the following sentence.

Dividing by 10 moves the decimal point ....

Try these

$2.46 \div 100 =$ Note that 100 could be written 10^2
$25.638 \div 100 =$
$5\,694.3 \div 100 =$
$0.008\,94 \div 100 =$
$43 \div 100 =$

Complete the following sentence.

Dividing by 100 moves the decimal point ....

Finally try these.

2.46 ÷ 100 000 = Note that 100 000 could be written 10^5

25.638 ÷ 100 000 =

5 694.3 ÷ 100 000 =

0.008 94 ÷ 100 000 =

43 ÷ 100 000 =

Complete the following sentence.

Dividing by 100 000 moves the decimal point ....

We can generalise these rules for dividing a number by a power of ten.

> When **dividing** by a power of 10, move the decimal point to the left by the number of zeros in the power of 10.

To divide decimals:

- Estimate the answer as before.
- Write an equivalent expression so that you are dividing by a whole number.
- Divide as for whole numbers.
- Position the decimal point above the decimal point in the question.
- Check your answer against the estimate.

Example

Evaluate 11.6112 ÷ 2.36

Estimate:

11.6112 ≈ 10

2.36 ≈ 2

10 ÷ 2 = 5

Calculate:

11.6112 ÷ 2.36

= 11.6112 ÷ 2.36

Write an equivalent expression so as to divide by a whole number. Shift the decimal point 2 places to the right in 2.36 to make a whole number. Do the same to the 11.6112

= 1161.12 ÷ 236

```
           4 . 9 2
2 3 6 ) 1 1 6 1 . 1 2
          9 4 4
          2 1 7  1
          2 1 2  4
               4 7 2
               4 7 2
                   0
```

Divide as for whole numbers. Place the decimal point in the answer above the decimal point in the question. Do not bring the decimal point down underneath the question.

Check: Yes the answer is reasonable.
(Check on your calculator.)

Activity 2.29

1. Evaluate the following. Estimate your answers before you begin.

   (a) $0.672 \div 3.2$

   (b) $-8.326 \div 0.23$

   (c) $0.002\ 88 \div 0.012$

   (d) $-16.605 \div {}^{-}36.9$

   (e) $810 \div 40$

2. A piece of pipe 6 metres long is to be cut into pieces that are each 0.75 metres long. How many pieces will there be?
3. If $23.58 is paid per month on a car repair account, how long would it take to pay off a balance of $565.92?
4. A vial of a certain drug contains 12.5 millilitres. How many single doses of 1.25 millilitres can be given from this vial?
5. Companies wishing to export products to countries not using the metric system must supply measurements in imperial units (the system Australia used to use). To convert from centimetres to inches you divide by 2.54. How many inches are there in 280 centimetres?
6. For a $5.00 fee, a person can receive a flu shot at a community clinic. It costs the clinic $0.50 for the vaccine. If 154 people received flu shots on Monday, what was the profit for the clinic?

2.6.4 Converting between decimals and fractions

Sometimes it is convenient to work in fractions while at other times it may be more convenient to work in decimals. Although your calculator can work in fractions or decimals while the numbers are small, it converts to decimals once we start dealing with larger numbers. There are also instances where we would not normally talk in terms of fractions, for example we wouldn't say we had $\frac{3}{5}$ of a dollar, we would say we had 60 cents or $0.60.

It is important then to be able to convert from decimals to fractions and from fractions to decimals.

Let's convert from a fraction to a decimal.

Recall that a fraction is just another way of expressing a division. It is this knowledge that enables us to convert a fraction to a decimal.

Example

Convert $\frac{3}{5}$ to a decimal.

We know that $\frac{3}{5}$ means $3 \div 5$

We know that in a whole number the decimal point is located immediately after the last number. Placing zeros after the decimal point does not effect the number. When converting a fraction to a decimal it will be necessary to add some zeros.

$$\begin{array}{r} 0\,.\,6 \\ 5\overline{)3\,.\,0} \\ \underline{3\quad 0} \\ 0 \end{array}$$

Check this on the calculator: 3 ÷ 5 = The display should read 0.6

Note that when there are no whole numbers, a zero is always written in the unit's place.

Example

Express $\frac{-7}{11}$ as a decimal.

On the calculator: (−) 7 ÷ 1 1 =

The display should read −0.63636363 … Don't forget the negative sign in your answer

This number will continue like this indefinitely. We write this as $-0.\dot{6}\dot{3}$ and say *−0.63 recurring.*

If the decimal was 56.478 947 89 … we would write this $56.\dot{4}78\dot{9}$ with a dot over the first and last digit in the recurring part of the decimal.

Often it will be necessary to round off the answer your calculator gives if you cannot see a recurring pattern. In this case the fraction gives you an **exact** answer while the decimal is an **approximation.**

Let's now look at the case where we have a decimal and we require it to be written as a fraction.

Consider the following:

$0.3 = 3 \text{ tenths} = \frac{3}{10}$

You can see that there was one decimal place in the decimal and there is one zero in the denominator of the fraction.

$0.37 = 37 \text{ hundredths} = \frac{37}{100}$

This time there are two decimal places in the decimal and two zeros in the denominator of the fraction.

We can generalise this:

- Write down the number without the decimal point.
- Count the number of decimal places, and put this many zeros after a 1 in the denominator.
- Simplify the fraction if possible.

Example

Convert 0.45 to a fraction.

0.45	Write down the number without the decimal point.
$= \frac{45}{100}$	Count the number of decimal places, and put this many zeros after a 1 in the denominator.
$= \frac{9}{20}$	Simplify.

Example

Convert −5.25 to a fraction.

-5.25	Write down the number without the decimal point.
$= -\frac{525}{100}$	Count the number of decimal places, and put this many zeros after a 1 in the denominator.
$= \frac{-21}{4}$	Simplify.

$= -5\frac{1}{4}$

Activity 2.30

1. Convert the following fractions to decimals.

   (a) $\frac{1}{8}$

   (b) $-\frac{1}{4}$

   (c) $-\frac{7}{10}$

   (d) $\frac{1}{3}$

   (e) $\frac{13}{7}$

   (f) $-\frac{25}{11}$

   (g) $1\frac{3}{4}$

2. Convert the following decimals to fractions. Express your answer in simplest Irrational numbers form.

   (a) 0.7

   (b) 0.8

   (c) 0.35

   (d) −0.58

   (e) 0.075

   (f) −0.625

   (g) 4.02

So far we have looked at decimals that **terminate** (e.g. 0.25, 1.648, 458.1) and decimals that **recur** (e.g. 0.33333…, 0.636363…, 56.666666…). There is one other type of decimal that neither terminates nor recurs. We call this type of decimal an **irrational number**.

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{7}$. We can find an approximate answer for these using the calculator, but in reality the decimal keeps going forever.

$\sqrt{2} = 1.414213562$..............

$\sqrt{3} = 1.732050808$..............

$\sqrt{7} = 2.645751311$..............

The approximation that the calculator gives, enables us to position the irrational number on the number line.

Before we move on and have a final look at the number line let's look at one particular irrational that you may be familiar with. It is the number π (the Greek letter pi).

We will use π in later modules when working with circles. Your calculator will give you an approximation for π.

$\pi = 3.141592654$..............

In the past you may have used $\frac{22}{7}$ as an **approximation** for π. On your calculator get a value for $\frac{22}{7}$

$\frac{22}{7} = 3.142857143$..............

As you can see, it is accurate only to the second decimal place.

For our calculations in these modules we will use the π key on the calculator for greatest accuracy.

Let's now take a final look at the number line.

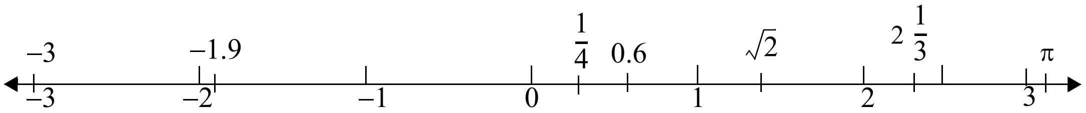
Example

Plot the following points on the number line.

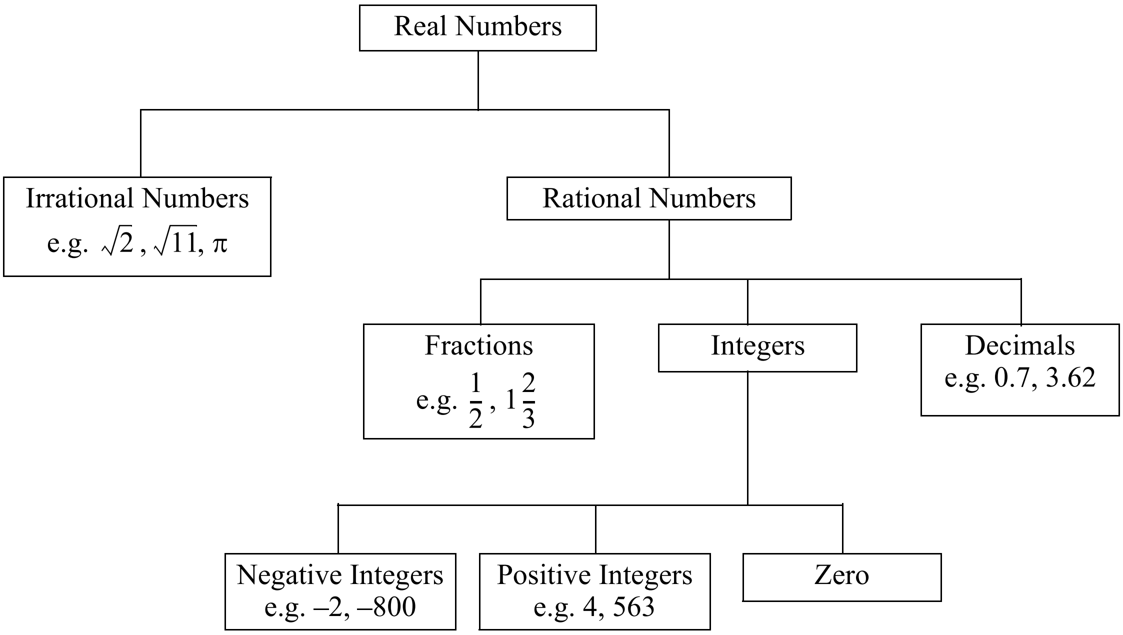
$\frac{1}{4}$, 0.6, $2\frac{1}{3}$, π, −1.9, 0, −3, $\sqrt{2}$

To work out the position on the number line of each of these points it is convenient to convert them all to decimal approximations.

$\frac{1}{4} = 0.25$ $\quad 0.6 \quad$ $2\frac{1}{3} \approx 2.33$ $\quad \pi \approx 3.14 \quad$ $-1.9 \quad$ $0 \quad$ $-3 \quad$ $\sqrt{2} \approx 1.41$



You can see above all the different types of numbers we have used in this module. Together, all these numbers are called the **real numbers** and are best described in the picture below.



You have now not only studied a range of different numbers, but also how to calculate with them. When you are confident with these skills, have a go at the following questions which have been taken from real-life situations or situations you might encounter in your future tertiary studies.

Assignment

You should now be ready to attempt questions 5 and 6 of Assignment 1A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

2.7 A taste of things to come

1. Following is a bankcard statement.

The following is a bankcard account statement for

J Citizen
Grubby Ave
Toowoomba 4350

Account number		Annual percentage rate **Cash advances:** 16.450%	Daily percentage rate	**Opening balance A$**
Credit limit $ 4 000	Credit available	Annual percentage rate **Purchases** 16.450%	Daily percentage rate	**$1 246.73**

Date	Reference	Transaction details			Amount A$	
08 JAN97		TARGET 5041	TOOWOOMBA	AU	57.60	DB
13 JAN97		SHELL SUPERBARGAIN	TOOWOOMBA	AU	53.00	DB
16 JAN97		K MART 1029	TOOWOOMBA	AU	78.00	CR
21 JAN97		FOSSEYS 212	ROCKHAMPTON	AU	45.85	DB
26 JAN97		BIG W 0260	TOOWOOMBA	AU	57.51	DB
28 JAN97		CALTEX	TOOWOOMBA	AU	40.00	DB
29 JAN97		K MART 1029	TOOWOOMBA	AU	16.60	DB
03 FEB97		PAYMENT – THANK YOU			100.00	CR
04 FEB97		WYALLA PLAZA DAY NIGHT	TOOWOOMBA	AU	41.10	DB
05 FEB97		CREDIT CHARGE – PURCHASES			19.51	
05 FEB97		CONTRACT STAMP DUTY			.83	

Opening balance	Total credits (CR)	Total debits (DB)	Credit and other charges	**Closing balance A$**

Past due	Due date	Min.payment due	Payment record	Date paid	Amount paid
.00	**28 FEB97**				$

You may not have a bankcard yourself but this is an important exercise in reading any type of financial statement. You should look carefully over the different sections of the statement. Many things are self explanatory but I will draw your attention to a few items.

DB after an amount means debit. The debits are added to your balance owing.

CR after an amount means credit. The credits are subtracted from your balance owing. They are payments that reduce the balance owing.

The last two items on the statement are charges that the bank makes on your account. You should now be in a position to answer the following questions.

(a) What is the credit limit on this account?
What do you think this means?

(b) By what date should the closing balance or the minimum payment have been made?

(c) Fill out the first row at the bottom of the statement. That is, Opening balance, Total credits (CR), Total debits (DB), credit and other charges and the Closing balance.

We will look further at this statement in a future module.

2. In 1987 the forensic toxicology group of the Queensland Government Chemical Laboratory performed drug analysis on samples of liver, stomach, blood and urine from 374 corpses involved in post mortem examinations. The numbers of times various drugs were detected at toxic levels are given in the following table.

Drugs detected at toxic levels in post mortem specimens

Drug	**Single drug**	**With alcohol**	**More than one drug**	**Subtotal**	**Total**
Analgesics					
Codeine	–	–	5	5	
Paracetamol	–	–	4	4	
Propoxyphene	1	1	2	4	13
Barbiturates					
Amylobarbitone	1	–	1	2	
Pentobarbitone	1	1	3	5	
Quinalbarbitone	1	–	1	2	9
Chlorpromazine	–	1	5	6	6
Chloroquin	7	1	–	8	8
Digoxin	–	–	1	1	1
Metroprolol	–	–	1	1	1
Narcotic analgesics					
Methadone	–	–	6	6	
Morphine	2	–	1	3	9
Orphenadrine	–	–	1	1	1
Pancuronium	–	–	1	1	1
Propranolo	–	–	2	2	2
Theophylline	1	1	–	2	2
Trichloroethanol	–	1	2	3	3
Tricyclic antidepressants					
Amitriptyline	–	–	5	5	
Clomipramine	–	–	1	1	
Dothiepin	–	–	2	2	
Doxepin	1	2	2	5	13
Verapamil	–	1	–	1	1
TOTAL	15	9	46	–	70

(Source: Moore, C 1988, 'Forensic toxicology', ***Australian Science Mag***., no. 4, Darling Downs Institute Press, Toowoomba.)

(a) What fraction of all corpses showed toxic levels of drugs?

(b) What fraction (of corpses showing toxic drug levels) showed toxic levels of:
  (i) analgesics
  (ii) barbiturates
  (iii)pentobarbitone?

(c) What fraction of all corpses showed the presence of more than one drug?

(d) What fraction of all corpses did not show the presence of any of the drugs in the table?

3. Following is a table showing information obtained from a breast screening clinic.

Lower-risk group shouldn't let lump go unnoticed

Age	**Group size for 1 to develop breast cancer per year**
40	1 041
45	653
50	610
55	515
60	440
65	392
70	382

(a) Show several people the information in the chart and ask them the following question.

According to the information given in the table, is it true that as a person gets older, the chance of developing breast cancer lessens?

(b) What is the correct interpretation of this information? To establish this, write the chance of getting breast cancer for each age group as a fraction. For example, for a woman aged 40 the chance of getting breast cancer is 1/1 041. That is, she has one chance out of 1 041 that she will develop breast cancer.

Were most people you asked able to give the correct interpretation of the information?

(c) If you were asked to write a short article for a newspaper to accompany the above table, you could use some of the information that you have worked out above. Write a few sentences now, that could be included in an article to accompany this table.

Congratulations, you have now completed the second mathematics module. You should check that you have mastered all the objectives listed in the introduction to this module. The post-test is included here to help you do this.

Now might be a good time to look at your summary of new ideas, words and symbols. Would you be able to explain all these to a friend?

When you feel confident, complete the assignment, together with its learning diary.

2.8 Post-test

1. Evaluate the following expressions **without** a calculator. Show all your working. Estimate your answer before commencing and check your answer with your calculator.

   (a) $765 \div 15 + 822$

   (b) $89 + 21 - 48 \times 23$

   (c) $591 \times 376 + 523$

   (d) $895(622 + 479)$

   (e) $4\,763 + 395 \div 5 \times 16$

2. Evaluate the following expressions on your calculator. Estimate your answer before commencing. Round your answers to two decimal places if necessary.

   (a) $(8.47)^2 \times 6.23$

   (b) $\sqrt{\dfrac{584}{73.8}}$

   (c) $(\sqrt{8.25})^2 - 85.72$

   (d) $\dfrac{6.4 \times 34}{12.4}$

   (e) $(48 \div 8.4)^3$

3. The following table contains common fractions and decimals that you should become very familiar with. Complete the table.

Fraction	$\frac{1}{8}$		$\frac{1}{4}$	$\frac{1}{3}$		$\frac{2}{3}$	
Decimal		0.2			0.5		0.75

4.

   (a) Mark the following numbers on a number line.

   $0.2^2,\ -3\frac{1}{2},\ \pi,\ -\sqrt{3},\ 3.8,\ \frac{4}{5}$

(b) Using your calculator find the sum of the numbers in part (a). Round your answer to the nearest thousandth.

5. The following table shows the weight fluctuations of a person on a weight loss program.

Week	1	2	3	4	5	6	7	8
loss/gain (kilograms)	−2.6	−1.2	+0.4	−0.9	+1.5	−2.7	−0.1	+0.4

(a) If they started the program weighing 94.7 kilograms, what was their weight after the eight weeks shown?

(b) What was the total amount to weight lost?

(c) If this person's ideal weight is 68 kilograms, how much more weight do they need to lose?

6. Toowoomba City Council's 1996/97 annual rates for property owners include the following charges.

• General Rate on unimproved land value.	
Residential	1.729 cents per $ value
Central Business District	2.029 cents per $ value
• Park and Bushland charge	$15.00
• Water Access charge	$235.00
• Sewerage Charge	
Domestic (any number of pedestals)	$134.00
Commercial First pedestal	$134.00
Commercial Other pedestal	$124.00 each
• Cleaning Charges	
Domestic	$70.00
Commercial	$170.00
• Fire Levy	$95.20

(a) Toowoomba City Council issues rate notices twice a year. Calculate the half yearly rates and charges due to Council on a residential property with an unimproved land value of $42 500.

(b) Calculate the half yearly rates and charges due to Council on a commercial property with 4 pedestals in the central business district with an unimproved land value of $82 540.

7. Following are the 1995 details of who moved where in Australia.

Arrived in	**Left NSW for**	**Left Vic for**	**Left Qld for**	**Left SA for**	**Left WA for**	**Left Tas for**	**Left NT for**	**Left ACT for**	**TOTAL**
Qld	53 110	31 069		8 900	7 548	4 147	6 205	3 894	114 873
NSW		24 182	34 184	6 445	6 770	2 442	2 532	10 496	87 051
Vic	19 442		14 628	6 776	5 191	3 125	2 052	2 231	53 445
WA	7 669	7 472	6 618	3 759		1 626	3 587	1 006	31 737
SA	5 682	7 446	4 772		2 542	755	2 783	724	24 704
ACT	10 425	2 603	2 876	1 132	985	319	739		19 079
NT	3 430	2 922	4 805	3 202	2 606	443		594	18 002
Tas	2 272	2 699	2 216	1 019	1 173		452	311	10 142
TOTAL									

(a) To which state or territory did the most people move?

(b) Find the total number of people that left each of the states and territories. Which state or territory had the greatest number of people leave?

(c) Nett migration means the difference between the number of people coming in to the state or territory and the number of people leaving. By completing the following table decide which states or territories had a negative nett migration.

State	**Arrived**	**Departed**	**Nett migration**
Qld	114 873		
NSW	87 051		
Vic	53 445		
WA	31 737		
SA	24 704		
ACT	19 079		
NT	18 002		
Tas	10 142		

(d) The population of which state or territory remained least changed after arrivals and departures had been accounted for?

(e) What fraction of the total number of people who left NSW, went to South Australia?

(f) What fraction of the total number of people arriving in Western Australia came from Queensland?

2.9 Solutions

Solutions to activities

Activity 2.1

8.	(a)	T	2.	(a)	6 < 12
	(b)	F		(b)	74 > 56
	(c)	T		(c)	127 > 89
	(d)	T		(d)	17 < 34
	(e)	T		(e)	41 > 28

Activity 2.2

	Number	**To nearest ten**	**To nearest hundred**	**To nearest thousand**
(a)	2 575	2 580	2 600	3 000
(b)	324	320	300	0
(c)	105	110	100	0
(d)	26 897	26 900	26 900	27 000
(e)	5 502 471	5 502 470	5 502 500	5 502 000

Activity 2.3

(a)	30	(e)	200 000
(b)	50	(f)	900
(c)	100	(g)	100 000
(d)	400	(h)	60 000

Activity 2.4

1. (a) 58 + 61

Estimate	Calculate	Check
58 ≈ 60	58	
61 ≈ 60	+ 61	119 is close to our estimate.
60 + 60 = 120	119	

(b) 25 + 956 + 32

Estimate	Calculate	Check
25 ≈ 30	25	
956 ≈ 1 000	956	1 013 is close to our estimate.
32 ≈ 30	+ 32	
	1 1	
30 + 1 000 + 30 = 1 060	1 013	

(c) 750 + 2 305 + 10 + 9 + 315

Estimate	Calculate	Check
750 ≈ 800	750	
2 305 ≈ 2 000	2 305	
	10	
	9	3 389 is close to our estimate.
315 ≈ 300	+ 315	
	1 1	
800 + 2 000 + 300 + 10 + 9 = 3 119	3 389	

(d) 658 + 0

You would know that adding zero does not change a number so there is no need to estimate in this case.

658 + 0 = 658

2.

Sunday	5
Monday	3
Tuesday	2
Friday	6
Saturday	9

Total admitted = 5 + 3 + 2 + 6 + 9 = 25 children

3. Total attendance = 10 428 + 8 922 + 7 431 + 9 647 people

Estimate	Calculate	Check
10 428 ≈ 10 000	10 428	
8 922 ≈ 9 000	8 922	This answer is close to our estimate.
7 431 ≈ 7 000	7 431	
9 647 ≈ 10 000	+ 9 647	
	2 2 1 1	
10 000 + 9 000 + 7 000 + 10 000 = 36 000	36 428	

Total attendance was 36 428 people

4. David's lunch = 2 103 + 1 714 + 1 148 + 18 kJ

Estimate	Calculate	Check
2 103 ≈ 2 000	2 103	
1 714 ≈ 2 000	1 714	
1 148 ≈ 1 000	1 148	This answer looks reasonable.
18 ≈ 20	+ 18	
	2	
2 000 + 2 000 + 1 000 + 20 = 5 020	4 983	

David's lunch contained 4 983 kJ

Activity 2.5

1. (a) 759 – 326

Estimate	Calculate	Check
759 ≈ 800	759	
326 ≈ 300	– 326	This answer looks reasonable.
800 – 300 = 500	433	

(b) 39 200 – 14 125

Estimate	Calculate	Check
	9 10	
	$11\cancel{0}$	
39 200 ≈ 40 000	$39\ \cancel{2}\cancel{0}\cancel{0}$	
14 125 ≈ 10 000	– 14 125	This answer looks reasonable
40 000 – 10 000 = 30 000	25 075	

(c) 126 430 – 2 472

Estimate	Calculate	Check
	5 13	
	$\cancel{3}$12	
	$\cancel{2}$10	
126 430 ≈ 100 000	12$\cancel{6}$ $\cancel{4}\cancel{3}$0	
2 472 ≈ 2 000	– 2 472	This answer is reasonable.
100 000 – 2 000 = 98 000	123 958	

(d) 37 – 0

You would know that subtracting zero does not change a number so there is no need to estimate in this case

37 – 0 = 37

2. The cost of Mary's items = $14 + $27 + $16 + $7

Estimate	Calculate	Check
14 ≈ 10	14	
27 ≈ 30	27	
16 ≈ 20	16	This answer is reasonable.
7 = 7	+ 7	
	2	
10 + 30 + 20 + 7 = 67	64	

Mary's change = 100 – 64

Estimate	Calculate	Check
	9 10	
100 ≈ 100	$\cancel{1}\cancel{0}\cancel{0}$	
64 ≈ 60	– 64	This answer is reasonable
100 – 60 = 40	36	

Mary should receive $36 change

3. Amount of water to add = 3 500 – 542 mL

Estimate	Calculate	Check
	2 14 9 10	
	$\cancel{4}\cancel{10}$	
3 500 ≈ 4 000	$\cancel{3}$ $\cancel{5}\cancel{0}\cancel{0}$	This answer looks reasonable.
542 ≈ 500	– 542	
4 000 – 500 = 3 500	2 958	

The cleaner should add 2 958 mL of water

4. (a)

Name	**Gross Pay**	**Tax**	**Superannuation**	**Union Fees**	**Take Home Pay**
Adams J	$500	$105	$30	$2	$363
Bull P	$1 200	$407	$74	$2	$717
Filbee Y	$678	$169	$41	$2	$466
Hand I	$893	$261	$54	$2	$576
Ruse K	$560	$127	$34	$2	$397
Totals	$3 831	$1 069			

(b) The total wages bill for these employees is $3 831.

(c) The tax to be sent to the taxation office is $1 069.

5. (a) Gold Coast to Townsville needs 1 450 points

(b) Brisbane to Sydney needs 850 points

(c) Sydney to Melbourne needs 850 points

(d) Brisbane to Melbourne direct needs 1 450 points
Linda used 850 + 850 = 1 700 points
The direct flight saves 1 700 – 1 450 points
Linda would have saved 250 points

Activity 2.6

(a) $-5 < 3$

(b) $-1 > -4$

(c) $3 > -1$

(d) $-65 < -34$

(e) $-125 > -420$

Activity 2.7

1. (a) 9×45

Estimate	Calculate		Check
$9 = 9$ $45 \approx 50$ $9 \times 50 = 450$	45 × 9 4 365 1 405	It is an easier calculation if we put the smaller number on the bottom.	This answer looks reasonable.

(b) 93 × 72

Estimate	Calculate	Check
93 ≈ 90	93	
72 ≈ 70	× 72	This answer looks reasonable.
90 × 70 = 6 300	186	
	2	
	6 310	
	6 696	

(c) 195 × 24

Estimate	Calculate	Check
195 ≈ 200	195	
24 ≈ 20	× 24	This answer looks reasonable.
	3 2	
200 × 20 = 4 000	460	
	1 1	
	2 800	
	1	
	4 680	

(d) 1 952 × 346

Estimate	Calculate	Check
1 952 ≈ 2 000	1 952	
346 ≈ 300	× 346	This answer looks reasonable.
	5 3 1	
2 000 × 300 = 600 000	6 402	
	3 2	
	46 080	
	2 1	
	375 600	
	1 2 1	
	675 392	

(e) 589 × 40

Estimate	Calculate		Check
589 ≈ 600	589	In this case it is not necessary to write out the row of zeros. We can move straight to multiplying by the 4.	This answer looks reasonable.
40 = 40	× 40		
	3 3		
600 × 40 = 24 000	20 260		
	23 560		

2. Sally earns $14 × 38

	Estimate	Calculate	Check
14	≈ 10	38	
38	≈ 40	$\underline{\times 14}$	Yes, this looks reasonable.
		^{3}	
10×40	$= 400$	122	
		380	
		^{1}	
		$\overline{532}$	

Sally earns $532 for the week.

3. Total weight = 36 × 22 kilograms

	Estimate	Calculate	Check
36	≈ 40	36	
22	≈ 20	$\underline{\times 22}$	Yes, looks reasonable.
		^{1}	
40×20	$= 800$	62	
		^{1}	
		$\underline{620}$	
		792	

Total weight of concrete blocks is 792 kilograms.

4. (a) Saving = $45 – $26

	Estimate	Calculate	Check
		$^{3\,15}$	
45	≈ 50	$\cancel{4}\cancel{5}$	
26	≈ 30	$-\underline{26}$	Yes, looks reasonable.
$50 - 30$	$= 20$	19	

Jeremy would save $19

(b) Saving on 5 shirts = 5 × $19

	Estimate	Calculate	Check
5	$= 5$	19	
19	≈ 20	$\underline{\times 5}$	Yes, looks reasonable.
		^{4}	
5×20	$= 100$	$\underline{55}$	
		95	

Jeremy would save $95 on 5 shirts.

5. (a) Passengers travelling to Melbourne $= 36 - 14 = 22$

   (b) After passengers unloaded in Sydney: total $= 36 - 14 = 22$
   After passengers loaded in Sydney: total $= 22 + 23 = 45$
   So 45 passengers get off the coach in Melbourne.

Activity 2.8

1. (a) $3^3 = 3 \times 3 \times 3 = 27$
   (b) $2^4 = 2 \times 2 \times 2 \times 2 = 16$
   (c) $5^4 = 5 \times 5 \times 5 \times 5 = 625$
   (d) $4^1 = 4$
   (e) $9^2 = 9 \times 9 = 81$
   (f) $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7\,776$

2. (a) $4^2 = 16$
   (b) $2^5 = 32$
   (c) $5^2 = 25$
   (d) $3^4 = 81$

3. Total reading time $= 7$ children $\times$ 7 minutes $\times$ 7 days

$$= 7^3$$

$$= 343$$

The Heptane children spend 343 minutes reading.

Activity 2.9

(a) $\sqrt{81} = 9$
(b) $\sqrt{36} = 6$
(c) $\sqrt{25} = 5$
(d) $\sqrt{64} = 8$
(e) $\sqrt{100} = 10$

Activity 2.10

1. (a) $1\,204 \div 4$

Estimate	Calculate	Check
$1\,204 \approx 1\,000$ $4 = 4$ $1\,000 \div 4$ is between 200 and 300	$\begin{array}{r} 3\,0\,1 \\ 4\overline{)1\,2\,0\,4} \\ \underline{1\,2} \\ 0\,0 \\ \underline{0\,0} \\ 0\,4 \\ \underline{4} \\ 0 \end{array}$	Yes, looks reasonable.

(b) 432 ÷ 12

Estimate	Calculate	Check
432 ≈ 400 12 ≈ 10 400 ÷ 10 = 40	$\begin{array}{r} 36 \\ 12\overline{)432} \\ \underline{36} \\ 72 \\ \underline{72} \\ 0 \end{array}$	Yes, looks reasonable.

(c) 10 608 ÷ 26

Estimate	Calculate	Check
10 608 ≈ 10 000 26 ≈ 30 10 000 ÷ 30 is between 300 and 400	$\begin{array}{r} 408 \\ 26\overline{)10608} \\ \underline{104} \\ 20 \\ \underline{00} \\ 208 \\ \underline{208} \\ 0 \end{array}$	Yes, looks reasonable.

(d) 3 612 ÷ 42

Estimate	Calculate	Check
3 612 ≈ 4 000 42 ≈ 40 4 000 ÷ 40 = 100	$\begin{array}{r} 86 \\ 42\overline{)3612} \\ \underline{336} \\ 252 \\ \underline{252} \\ 0 \end{array}$	Yes, looks reasonable.

2. Each part = $4 500 ÷ 3

Estimate	Calculate	Check
4 500 ≈ 5 000 3 = 3 5 000 ÷ 3 is between 1 000 and 2 000	$\begin{array}{r} 1500 \\ 3\overline{)4500} \\ \underline{3} \\ 15 \\ \underline{15} \\ 0 \end{array}$	Yes, looks reasonable.

Each child receives $1 500

3. Amount of mince for each meal = 2 700 ÷ 6

Estimate	Calculate	Check
2 700 ≈ 3 000 6 = 6 3 000 ÷ 6 = 500	$\begin{array}{r} 4\,5\,0 \\ 6\overline{)\,2\,7\,0\,0} \\ \underline{2\,4} \\ 3\,0 \\ \underline{3\,0} \\ 0 \end{array}$	Yes, looks reasonable.

Joseph freezes 450 grams for each meal.

4. Daily rate is $31 025 ÷ 365

Estimate	Calculate	Check
31 025 ≈ 30 000 365 ≈ 400 30 000 ÷ 400 is between 70 and 80	$\begin{array}{r} 8\,5 \\ 365\overline{)3\,1\,0\,2\,5} \\ \underline{2\,9\,2\,0} \\ 1\,8\,2\,5 \\ \underline{1\,8\,2\,5} \\ 0 \end{array}$	Yes, looks reasonable.

Weekly rate is $85 × 7

Estimate	Calculate	Check
85 ≈ 90 7 = 7 90 × 7 = 630	$\begin{array}{r} 85 \\ \underline{\times\,7} \\ {\scriptstyle 3} \\ \underline{565} \\ 595 \end{array}$	Yes, looks reasonable

Your weekly wage would be $595

5. Since 8 minutes = 480 seconds Keely’s ride lasts 480 ÷ 32 revolutions

Estimate	Calculate	Check
480 ≈ 500 32 ≈ 30 500 ÷ 30 is between 10 and 20	$\begin{array}{r} 1\,5 \\ 32\overline{)4\,8\,0} \\ \underline{3\,2} \\ 1\,6\,0 \\ \underline{1\,6\,0} \\ 0 \end{array}$	Yes, looks reasonable.

The merry-go-round revolved 15 times.

Activity 2.11

1. (a) $-7 + {}^{-}16 = -(7 + 16) = -23$ (b) $-4 + {}^{-}9 = -(4 + 9) = -13$

   (c) $-25 + {}^{-}12 = -(25 + 12) = -37$ (d) $-456 + {}^{-}32 = -(456 + 32) = -488$

   (e) $-3 + {}^{-}1\,245 = -(3 + 1\,245) = -1\,248$

2. The submarine starts at –10 metres. The submarine dives to a depth of $-10 + {}^{-}20$ metres.

   $-10 + {}^{-}20 = -(10 + 20) = -30$

   The submarine comes to rest 30 metres below sea level.

3. Dan’s cheque account balance is –\$110.
   After another withdrawal the balance is $-110 + {}^{-}43$.

   Dan’s balance is $-110 + {}^{-}43 = -(110 + 43) = -153$.

   Dan’s account is overdrawn by \$153

Activity 2.12

1. (a) $-5 + 3 = -(5 - 3) = -2$ (b) $-56 + 78 = +(78 - 56) = 22$

   (c) $456 + {}^{-}67 = +(456 - 67) = 389$ (d) $89 + {}^{-}567 = -(567 - 89) = -478$

   (e) $-1\,789 + 1\,674 = -(1\,789 - 1\,674) = -115$

2. Nami still owes \$25 + $^{-}$\$5

   $25 + {}^{-}5 = +(25 - 5) = 20$

   Nami owes \$20

3. Submarine was first at –37 metres. After rising its position is –37 + 23 metres.

   $-37 + 23 = -(37 - 23) = -14$

   The submarine’s new depth is 14 metres below sea level.

4. Let Australia’s starting position be –134 runs.

   (a) After opening partnership Australia is –134 + 76 runs.

   $-134 + 76 = -(134 - 76) = -58$

   Australia is still 58 runs behind.

(b) At the end of the innings Australia is $-134 + 475$ runs.

$-134 + 475 = +(475 - 134) = 341$

At the end of the second innings Australia is 341 runs in front.

5. We will make descent negative and ascent (rising) positive.

The lift moves as follows:

$-3 + 6 + {}^{-}8 = 3 + {}^{-}8 = -5$

This lift is 5 floors below its starting position.

Activity 2.13

1. (a) $-45 - 89 = -45 + {}^{-}89 = -(45 + 89) = -134$

(b) $456 - 765 = 456 + {}^{-}765 = -(765 - 456) = -309$

(c) $-376 - {}^{-}46 = -376 + 46 = -(376 - 46) = -330$

(d) $73 - {}^{-}34 = 73 + 34 = 107$

(e) $567 - 345 = 222$

(f) $-498 - {}^{-}587 = -498 + 587 = +(587 - 498) = 89$

(g) $-45 - 238 = -45 + {}^{-}238 = -(45 + 238) = -283$

(h) $1\,234 - {}^{-}890 = 1\,234 + 890 = 2\,124$

(i) $-56 - 4\,693 = -56 + {}^{-}4\,693 = -(56 + 4\,693) = -4\,749$

(j) $567 - {}^{-}780 = 567 + 780 = 1\,347$

2. (a) $13 - \boxed{8} = 5$ (b) $7 - \boxed{9} = -2$

(c) $-3 - \boxed{3} = -6$ (d) $7 - \boxed{{}^{-}1} = 8$

(e) $\boxed{1} - 3 = -2$ (f) $\boxed{-6} - 3 = -9$

3. London $11 - 8 = 3$

Moscow $8 - {}^{-}1 = 8 + 1 = 9$

New York $3 - {}^{-}4 = 3 + 4 = 7$

Helsinki $-2 - {}^{-}6 = -2 + 6 = +(6 - 2) = 4$

City	**Maximum °C**	**Minimum °C**	**Difference °C**
London	11	8	3
Moscow	8	–1	9
New York	3	–4	7
Helsinki	–2	–6	4

4. Man dived 20 – 34 metres

   $20 - 34 = -(34 - 20) = -14$

   The man levelled out 14 metres below the top of the lake.

5. Bill beat Peter by $23 - {}^{-}4$ points

   $23 - {}^{-}4 = 23 + 4 = 27$

   Bill beat Peter by 27 points.

Activity 2.14

1. (a) $-3 \times 5 = -(3 \times 5) = -15$ (b) $4 \times {}^{-}7 = -(4 \times 7) = -28$

   (c) $-6 \times {}^{-}8 = +(6 \times 8) = 48$ (d) $-12 \times 6 = -(12 \times 6) = -72$

   (e) $-6 \times 0 = 0$

   (f) $-34 \times 29 = -(34 \times 29)$
   $= -986$

   Estimate: $34 \approx 30$
   $29 \approx 30$
   $30 \times 30 = 900$

   (g) $37 \times {}^{-}87 = -(37 \times 87)$
   $= -3\ 219$

   Estimate: $37 \approx 40$
   $87 \approx 90$
   $40 \times 90 = 3\ 600$

   (h) $-104 \times 6 = -(104 \times 6)$
   $= -624$

   Estimate:$104 \approx 100$
   $6 = 6$
   $100 \times 6 = 600$

   (i) $789 \times {}^{-}45 = -(789 \times 45)$
   $= -35\ 505$

   Estimate:$789 \approx 800$
   $45 \approx 50$
   $800 \times 50 = 40\ 000$

   (j) $-67 \times {}^{-}45 = +(67 \times 45)$
   $= 3\ 015$

   Estimate: $67 \approx 70$
   $45 \approx 50$
   $70 \times 50 = 3\ 500$

2. The temperature today is $2 \times {}^{-}3° = -6°$C

3. The diver is at –2 metres. The sea bed is 5 times this depth.

   That is, $5 \times {}^{-}2 = -10$ metres

   The sea bed is at 10 metres.

Activity 2.15

1. (a) $-42 \div 6 = -(42 \div 6) = -7$

   (b) $64 \div {}^{-}8 = -(64 \div 8) = -8$

   (c) $-861 \div {}^{-}3 = +(861 \div 3) = 287$

   (d) $-12 \div 6 = -(12 \div 6) = -2$

   (e) $-6 \div 2 = -(6 \div 2) = -3$

   (f) $-32 \div 16 = -(32 \div 16) = -2$

   (g) $5\,642 \div {}^{-}91 = -(5\,642 \div 91)$
   $= -62$

   Estimate: $5\,642 \approx 6\,000$
   $91 \approx 90$
   $6\,000 \div 90$ is between 60 and 70

   (h) $-5\,688 \div 6 = -(5\,688 \div 6)$
   $= -948$

   Estimate: $5\,688 \approx 6\,000$
   $6 = 6$
   $6\,000 \div 6 = 1\,000$

   (i) $765 \div {}^{-}45 = -(765 \div 45)$
   $= -17$

   Estimate: $765 \approx 800$
   $45 \approx 50$
   $800 \div 50$ is between 10 and 20

   (j) $-135 \div {}^{-}45 = +(135 \div 45)$
   $= 3$

   Estimate: $135 \approx 100$
   $45 \approx 50$
   $100 \div 50 = 2$

Activity 2.16

1. (a) $7 \times 5 + 4$
   $= 35 + 4$
   $= 39$

   (b) $10 - 6 \times 7$
   $= 10 - 42$
   $= -32$

   (c) $6 + (3 - 9)$
   $= 6 + {}^{-}6$
   $= 0$

   (d) $-2 - 2 \times {}^{-}3$
   $= -2 - {}^{-}6$
   $= -2 + 6$
   $= 4$

   (e) $9 \div 3 \times 7 + 3$
   $= 3 \times 7 + 3$
   $= 21 + 3$
   $= 24$

   (f) $-3 \div {}^{-}1 + 9^2 \times {}^{-}2$
   $= -3 \div {}^{-}1 + 81 \times {}^{-}2$
   $= 3 + {}^{-}162$
   $= -159$

   (g) $27 - \sqrt{9} + 21 \div {}^{-}3$
   $= 27 - 3 + 21 \div {}^{-}3$
   $= 27 - 3 + {}^{-}7$
   $= 24 + {}^{-}7$
   $= 17$

   (h) $4 \times (2 + 5) \div (3 + 1)$
   $= 4 \times 7 \div 4$
   $= 28 \div 4$
   $= 7$

   (i) $(17 - 5^3) \div {}^{-}3 + 9 \times {}^{-}5$
   $= (17 - 125) \div {}^{-}3 + 9 \times {}^{-}5$
   $= -108 \div {}^{-}3 + 9 \times {}^{-}5$
   $= 36 + {}^{-}45$
   $= -9$

   (j) $(-5 \times {}^{-}4)^2 - 6 \div (-3 + \sqrt{4})$
   $= 20^2 - 6 \div (-3 + 2)$
   $= 400 - 6 \div {}^{-}1$
   $= 400 - {}^{-}6$
   $= 400 + 6$
   $= 406$

2. (a) $765 \div 15 + 822$

Estimate	Calculate	Check
$800 \div 20 + 800$	$765 \div 15 + 822$	
$= 40 + 800$	$= 51 + 822$	
$= 840$	$= 873$	Answer looks reasonable.

(b) $89 + 21 - 48 \times 23$

Estimate	Calculate	Check
$90 + 20 - 50 \times 20$	$89 + 21 - 48 \times 23$	
$= 90 + 20 - 1\ 000$	$= 89 + 21 - 1\ 104$	
$= 110 - 1\ 000$	$= 110 - 1\ 104$	
$\approx 100 - 1\ 000$	$= -994$	
$= -900$		Answer looks reasonable.

(c) $591 + 37^2 \times \sqrt{49}$

Estimate	Calculate	Check
$600 + 40^2 \times 7$	$591 + 37^2 \times \sqrt{49}$	
$= 600 + 1\ 600 \times 7$	$= 591 + 1\ 369 \times 7$	
$\approx 600 + 2\ 000 \times 7$	$= 591 + 9\ 583$	
$= 600 + 14\ 000$	$= 10\ 174$	
$= 14\ 600$		Answer looks reasonable.

(d) $4\ 763 + 395 \div 5 \times 16$

Estimate	Calculate	Check
$5\ 000 + 400 \div 5 \times 20$	$4\ 763 + 395 \div 5 \times 16$	
$= 5\ 000 + 80 \times 20$	$= 4\ 763 + 79 \times 16$	
$= 5\ 000 + 1\ 600$	$= 4\ 763 + 1\ 264$	
$= 6\ 600$	$= 6\ 027$	Answer looks reasonable.

(e) $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}\ ) + 61 \times 453$

Estimate: $(60 - 20^2 + (7 + 3 \times 80) - 13) + 60 \times 500$

$\approx (60 - 400 + (7 + 240) - 10) + 30\ 000$

$= (60 - 400 + 247 - 10) + 30\ 000$

$\approx (60 - 400 + 200 - 10) + 30\ 000$

$= -150 + 30\ 000$

$\approx -200 + 30\ 000$

$= 29\ 800$

Calculate: $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}\ ) + 61 \times 453$

$= (62 - 24^2 + (7 + 243) - \sqrt{169}\ ) + 61 \times 453$

$= (62 - 24^2 + 250 - \sqrt{169}\ ) + 61 \times 453$

$= (62 - 576 + 250 - 13) + 61 \times 453$

$= -277 + 27\ 633$

$= 27\ 356$

Check: Answer looks reasonable.

3. (a) Ahmid earns $32 \times \$14 + 8 \times \$28$

Estimate: $30 \times 10 + 8 \times 30$
$= 300 + 240$
$= 540$

Calculate: $32 \times 14 + 8 \times 28$
$= 448 + 224$
$= 672$

Ahmid earns \$672 for the week.

(b) Mary earns $8 \times \$28 + 4 \times 8 \times \$14 + 4 \times 2 \times \$21$

Estimate: $8 \times 30 + 4 \times 8 \times 10 + 4 \times 2 \times 20$
$= 240 + 320 + 160$
$\approx 200 + 300 + 200$
$= 700$

Calculate: $8 \times 28 + 4 \times 8 \times 14 + 4 \times 2 \times 21$
$= 224 + 448 + 168$
$= 840$

Mary earns \$840 for the week.

4. Coach's load = $43 \times (54 + 12)$ kilograms

Estimate: $40 \times (50 + 10)$
$= 40 \times 60$
$= 2\ 400$

Calculate: $43 \times (54 + 12)$
$= 43 \times 66$
$= 2\ 838$

Coach's usual load is 2 838 kilograms.

5. Number of people going Brisbane to Melbourne = $36 - 14 = 22$
Number of people going Brisbane to Sydney = 14
Number of people going Sydney to Melbourne = 23
Total amount paid = $22 \times \$120 + 14 \times \$75 + 23 \times \$68$

Estimate: $20 \times 100 + 10 \times 80 + 20 \times 70$
$= 2\ 000 + 800 + 1\ 400$
$= 4\ 200$

Calculate: $22 \times 120 + 14 \times 75 + 23 \times 68$
$= 2\ 640 + 1\ 050 + 1\ 564$
$= 5\ 254$

Total amount paid to bus company is \$5 254

Activity 2.17

1. (a) $2(5 + 7) = 2 \times 5 + 2 \times 7$
$= 10 + 14$
$= 24$

(b) $3(2 + 6) = 3 \times 2 + 3 \times 6$
$= 6 + 18$
$= 24$

(c)
$$\begin{aligned} 5(7-2) &= 5 \times 7 - 5 \times 2 \\ &= 35 - 10 \\ &= 25 \end{aligned}$$

(d)
$$\begin{aligned} -2(3+5) &= -2 \times 3 + {}^{-}2 \times 5 \\ &= -6 + {}^{-}10 \\ &= -16 \end{aligned}$$

(e)
$$\begin{aligned} -7(5+2) &= -7 \times 5 + {}^{-}7 \times 2 \\ &= -35 + {}^{-}14 \\ &= -49 \end{aligned}$$

(f)
$$\begin{aligned} -2(3-6) &= -2 \times 3 - {}^{-}2 \times 6 \\ &= -6 - {}^{-}12 \\ &= -6 + 12 \\ &= 6 \end{aligned}$$

2. (a) $4 \times 8 + 4 \times 6 = 4(8+6) = 4 \times 14 = 56$

   (b) $3 \times 6 - 3 \times 7 = 3(6-7) = 3 \times {}^{-}1 = -3$

   (c) $-5 \times 8 + {}^{-}5 \times 14 = -5(8+14) = -5 \times 22 = -110$

   (d) $6 \times 48 + 6 \times 57 = 6(48+57) = 6 \times 105 = 630$

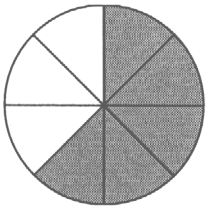
   (e) $-17 \times 8 - {}^{-}17 \times 61 = -17(8-61) = -17 \times {}^{-}53 = 901$

3.
$$\begin{aligned} \text{Number of pieces} &= 3 \times 6 + 3 \times 8 \\ &= 3(6+8) \\ &= 3 \times 14 \\ &= 42 \end{aligned}$$

There are 42 pieces of pizza.

Activity 2.18

1. (a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{3}{4}$ (d) $\frac{7}{10}$ (e) $\frac{1}{2}$

2. 

$\frac{5}{8}$ shaded

Activity 2.19

1. (a) $\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$ (b) $\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$

   (c) $\frac{-3}{4} = \frac{-3 \times 2}{4 \times 2} = \frac{-6}{8}$ (d) $\frac{-2}{7} = \frac{-2 \times 4}{7 \times 4} = \frac{-8}{28}$

   (e) $\frac{7}{12} = \frac{7 \times 5}{12 \times 5} = \frac{35}{60}$

2. (a) $\frac{9}{15} = \frac{\cancel{9}^{3}}{\cancel{15}_{5}} = \frac{3}{5}$ (dividing by 3)

   (b) $\frac{-3}{18} = \frac{-\cancel{3}^{-1}}{\cancel{18}_{6}} = \frac{-1}{6}$ (dividing by 3)

   (c) $\frac{8}{10} = \frac{\cancel{8}^{4}}{\cancel{10}_{5}} = \frac{4}{5}$ (dividing by 2)

   (d) $\frac{-12}{16} = \frac{-\cancel{12}^{-3}}{\cancel{16}_{4}} = \frac{-3}{4}$ (dividing by 4)

   (e) $\frac{30}{40} = \frac{3\cancel{0}}{4\cancel{0}} = \frac{3}{4}$ (We are dividing by 10 but in practice we just cancel the zeros.)

   (f) $\frac{2\,000}{6\,000} = \frac{2\,\cancel{000}}{6\,\cancel{000}} = \frac{\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{1}{3}$ (Cancel three zeros on the top and three on the bottom.)

3. (a) Strangled by trawler nets = $\frac{26}{75}$

   (b) Strangled by packaging bands = $\frac{15}{75} = \frac{1}{5}$

Activity 2.20

1. (a) $\frac{5}{3} = 1\frac{2}{3}$

   (b) $\frac{\cancel{45}^{9}}{\cancel{20}_{4}} = \frac{9}{4} = 2\frac{1}{4}$

   (c) $\frac{-16}{7} = -2\frac{2}{7}$

   (d) $\frac{\cancel{40}^{8}}{\cancel{25}_{5}} = \frac{8}{5} = 1\frac{3}{5}$

   (e) $\frac{-\cancel{35}^{-5}}{\cancel{14}_{2}} = \frac{-5}{2} = -2\frac{1}{2}$

2. (a) $3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{13}{4}$

   (b) $9\frac{2}{11} = \frac{9 \times 11 + 2}{11} = \frac{101}{11}$

   (c) $-5\frac{2}{3} = -\left(\frac{5 \times 3 + 2}{3}\right) = -\frac{17}{3}$

   (d) $102\frac{5}{12} = \frac{102 \times 12 + 5}{12} = \frac{1\,229}{12}$

   (e) $-21\frac{2}{5} = -\left(\frac{21 \times 5 + 2}{5}\right) = -\frac{107}{5}$

3. $5\frac{2}{3}$ cakes equals $\frac{5 \times 3 + 2}{3} = \frac{17}{3}$

   Therefore I have 17 thirds of cake.

4. Gary is $3\frac{5}{12} = \frac{3 \times 12 + 5}{12} = \frac{41}{12}$

Therefore, Gary is 41 twelfths or 41 months old.

Activity 2.21

1. (a) $$\begin{aligned} & \frac{3}{7} + \frac{1}{2} \\ &= \frac{6}{14} + \frac{7}{14} \\ &= \frac{13}{14} \end{aligned}$$

(b) $$\begin{aligned} & \frac{2}{3} - \frac{3}{4} \\ &= \frac{8}{12} - \frac{9}{12} \\ &= \frac{-1}{12} \end{aligned}$$

(c) $$\begin{aligned} & \frac{13}{15} + \frac{2}{5} \\ &= \frac{13}{15} + \frac{6}{15} \\ &= \frac{19}{15} \\ &= 1\frac{4}{15} \end{aligned}$$

(d) $$\begin{aligned} & \frac{-5}{6} + \frac{9}{10} \\ &= \frac{-25}{30} + \frac{27}{30} \\ &= \frac{2}{30} \\ &= \frac{1}{15} \end{aligned}$$

(e) $$\begin{aligned} & \frac{5}{9} - \frac{5}{6} + \frac{1}{2} \\ &= \frac{10}{18} - \frac{15}{18} + \frac{9}{18} \\ &= \frac{10 - 15 + 9}{18} \\ &= \frac{4}{18} \\ &= \frac{2}{9} \end{aligned}$$

(f) $$\begin{aligned} & 3\frac{1}{4} + 2\frac{1}{2} \\ &= \frac{13}{4} + \frac{5}{2} \\ &= \frac{13}{4} + \frac{10}{4} \\ &= \frac{23}{4} \\ &= 5\frac{3}{4} \end{aligned}$$

(g) $$\begin{aligned} & 5\frac{3}{10} - 4\frac{2}{5} \\ &= \frac{53}{10} - \frac{22}{5} \\ &= \frac{53}{10} - \frac{44}{10} \\ &= \frac{9}{10} \end{aligned}$$

(h) $$\begin{aligned} & -3\frac{7}{9} + 2\frac{1}{6} \\ &= \frac{-34}{9} + \frac{13}{6} \\ &= \frac{-68}{18} + \frac{39}{18} \\ &= \frac{-29}{18} \\ &= -1\frac{11}{18} \end{aligned}$$

(i) $$\begin{aligned} & 17\frac{1}{6} + 25\frac{1}{4} \\ &= \frac{103}{6} + \frac{101}{4} \\ &= \frac{206}{12} + \frac{303}{12} \\ &= \frac{509}{12} \\ &= 42\frac{5}{12} \end{aligned}$$

(j) $$\begin{aligned} & 2\frac{5}{7} - 3\frac{2}{9} \\ &= \frac{19}{7} - \frac{29}{9} \\ &= \frac{171}{63} - \frac{203}{63} \\ &= \frac{-32}{63} \end{aligned}$$

5. Jason ate $\frac{3}{8} = \frac{15}{40}$

Jenny ate $\frac{2}{5} = \frac{16}{40}$

It is easy to see now that Jenny ate the larger portion of the pizza.

6. Remaining medication = $1-\left(\frac{1}{3}+\frac{2}{5}\right)$

$$\begin{aligned}1-\left(\frac{1}{3}+\frac{2}{5}\right) &= 1-\left(\frac{5}{15}+\frac{6}{15}\right)\\ &= 1-\frac{11}{15}\\ &= \frac{15}{15}-\frac{11}{15}\\ &= \frac{4}{15}\end{aligned}$$

Therefore $\frac{4}{15}$ of the medication remains.

7. Fraction for entertainment $= 1-\left(\frac{1}{20}+\frac{1}{4}+\frac{1}{6}+\frac{2}{5}\right)$

$$\begin{aligned} &= 1-\left(\frac{3}{60}+\frac{15}{60}+\frac{10}{60}+\frac{24}{60}\right)\\ &= 1-\frac{52}{60}\\ &= \frac{60}{60}-\frac{52}{60}\\ &= \frac{\cancel{8}^{2}}{\cancel{60}_{15}}\\ &= \frac{2}{15}\end{aligned}$$

Therefore $\frac{2}{15}$ of Sean's income is left for entertainment.

8. Total amount of fill $= 2\frac{3}{4}+6\frac{2}{5}+4\frac{3}{8}+8\frac{1}{2}$ tonnes

$$\begin{aligned} &= \frac{11}{4}+\frac{32}{5}+\frac{35}{8}+\frac{17}{2}\\ &= \frac{110}{40}+\frac{256}{40}+\frac{175}{40}+\frac{340}{40}\\ &= \frac{881}{40}\\ &= 22\frac{1}{40}\end{aligned}$$

Amount of fill dumped was $22\frac{1}{40}$ tonnes.

9. Remaining length

$$\begin{aligned}
\text{Remaining length} &= 6 - \left(1\frac{3}{4} + 2\frac{1}{2} + \frac{3}{8}\right) \text{ metre} \\
&= 6 - \left(\frac{7}{4} + \frac{5}{2} + \frac{3}{8}\right) \\
&= 6 - \left(\frac{14}{8} + \frac{20}{8} + \frac{3}{8}\right) \\
&= 6 - \frac{37}{8} \\
&= \frac{48}{8} - \frac{37}{8} \\
&= \frac{11}{8} \\
&= 1\frac{3}{8}
\end{aligned}$$

The electrician has $1\frac{3}{8}$ metres of the tubing left.

Activity 2.22

1. (a)

$$\begin{aligned}
&\frac{5}{9} \times \frac{3}{20} \\
&= \frac{{}^{1}\cancel{5} \times \cancel{3}^{1}}{{}_{3}\cancel{9} \times \cancel{20}_{4}} \\
&= \frac{1 \times 1}{3 \times 4} \\
&= \frac{1}{12}
\end{aligned}$$

(b)

$$\begin{aligned}
&\frac{3}{8} \times \frac{7}{9} \\
&= \frac{{}^{1}\cancel{3} \times 7}{8 \times \cancel{9}_{3}} \\
&= \frac{1 \times 7}{8 \times 3} \\
&= \frac{7}{24}
\end{aligned}$$

(c)

$$\begin{aligned}
&-7\frac{3}{5} \times 4\frac{1}{2} \\
&= \frac{-38}{5} \times \frac{9}{2} \\
&= \frac{{}^{-19}\cancel{-38} \times 9}{5 \times \cancel{2}_{1}} \\
&= \frac{-19 \times 9}{5} \\
&= \frac{-171}{5} \\
&= -34\frac{1}{5}
\end{aligned}$$

(d)

$$\begin{aligned}
&-3\frac{3}{4} \times -7\frac{1}{20} \\
&= \frac{-15}{4} \times \frac{-141}{20} \\
&= \frac{{}^{-3}\cancel{-15} \times -141}{4 \times \cancel{20}_{4}} \\
&= \frac{-3 \times -141}{4 \times 4} \\
&= \frac{423}{16} \\
&= 26\frac{7}{16}
\end{aligned}$$

(e)

$$\begin{aligned}
&\frac{10}{15} \times \frac{-4}{9} \\
&= \frac{{}^{2}\cancel{10} \times -4}{{}_{3}\cancel{15} \times 9} \\
&= \frac{2 \times -4}{3 \times 9} \\
&= \frac{-8}{27}
\end{aligned}$$

2. (a) $\frac{3}{4} \times 256$ metres

$= \frac{3 \times \overset{64}{\cancel{256}}}{{}_{1}\cancel{4} \times 1}$

$= \frac{3 \times 64}{1 \times 1}$

$=$ 192 metres

(b) $\frac{4}{9} \times 594$ grams

$= \frac{4 \times \overset{66}{\cancel{594}}}{{}_{1}\cancel{9} \times 1}$

$= \frac{4 \times 66}{1 \times 1}$

$=$ 264 grams

(c) $\frac{3}{10} \times 45\ 000$ hectares

$= \frac{3 \times 45\ 00\cancel{0}}{1\cancel{0} \times 1}$

Remember just to cancel the zeros

$= 3 \times 4\ 500$

$=$ 13 500 hectares

3. Chung rested after $\frac{3}{5} \times 450$ kilometres

$\frac{3}{5} \times 450 = \frac{3 \times \overset{90}{\cancel{450}}}{{}_{1}\cancel{5} \times 1}$

$= 3 \times 90$

$= 270$

Chung rested after 270 kilometres.

4. The Smiths must drain $\frac{1}{3} \times 51\ 000$ litres

$\frac{1}{3} \times 51\ 000 = \frac{1 \times \overset{17\ 000}{\cancel{51\ 000}}}{{}_{1}\cancel{3} \times 1}$

$= 17\ 000$

The Smiths must drain 17 000 litres from the pool.

5. (a) Accommodation $= \frac{3}{5} \times \$1\ 850$

$= \frac{3 \times \overset{370}{\cancel{1\ 850}}}{{}_{1}\cancel{5} \times 1}$

$= 3 \times 370$

$= 1\ 110$

Accommodation cost $1 110

(b) Travelling

$$\begin{aligned} \text{Travelling} &= \frac{3}{10} \times \$1\,850 \\ &= \frac{3 \times 1\,85\cancel{0}}{1\cancel{0} \times 1} \\ &= 3 \times 185 \\ &= 555 \end{aligned}$$

Travelling cost $555

(c) Fraction for entertainment

$$\begin{aligned} \text{Fraction for entertainment} &= 1 - \left(\frac{3}{5} + \frac{3}{10}\right) \\ &= 1 - \left(\frac{6}{10} + \frac{3}{10}\right) \\ &= 1 - \frac{9}{10} \\ &= \frac{10}{10} - \frac{9}{10} \\ &= \frac{1}{10} \end{aligned}$$

Therefore $\frac{1}{10}$ was left for entertainment.

(d) Entertainment

$$\begin{aligned} \text{Entertainment} &= \frac{1}{10} \times \$1\,850 \\ &= \frac{1 \times 1\,85\cancel{0}}{1\cancel{0} \times 1} \\ &= \$185 \end{aligned}$$

Therefore $185 was spent on entertainment.

(e) The sum of accommodation, travelling and entertainment should give the total cost.

Check: $1 110 + $555 + $185 = $1 850

Yes it does so my calculations should be correct.

6. (a) Chris has multiplied by $\frac{8}{8}$ which equals 1 instead of $\frac{8}{1}$ which equals 8.

(b) You would need to explain the difference between $\frac{8}{8}$ and $\frac{8}{1}$

7. (a) Half were 'lifers'.

That is, $\frac{1}{2} \times 137 = \frac{1 \times 137}{2 \times 1} = \frac{137}{2} = 68\frac{1}{2}$

Now we cannot have half a person so we would round our answer to 69.

So 69 rebels were 'lifers'.

One third had 'fourteen years'.

That is, $\frac{1}{3} \times 137 = \frac{1 \times 137}{3 \times 1} = \frac{137}{3} = 45\frac{2}{3}$

Again, we cannot have two-thirds of a person, so we will round to 46.

So 46 rebels had fourteen year sentences.

(b) (i) Male convicts got 'one third less' than the naval standard ration. What does this mean?

It means that they got $1 - \frac{1}{3}$ or $\frac{2}{3}$ of the naval ration.

Beef $= \frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$ pounds Hardtack $= \frac{2}{3} \times 7 = \frac{14}{3} = 4\frac{2}{3}$ pounds

Pork $= \frac{2}{3} \times 2 = \frac{4}{3} = 1\frac{1}{3}$ pounds Cheese $= \frac{2}{{}_{1}\cancel{3}} \times \cancel{12}^{4} = 8$ ounces

Dried Peas $= \frac{2}{3} \times 2 = \frac{4}{3} = 1\frac{1}{3}$ pints Butter $= \frac{2}{{}_{1}\cancel{3}} \times \cancel{6}^{2} = 4$ ounces

Oatmeal $= \frac{2}{\cancel{3}} \times \cancel{3} = 2$ pints Vinegar $= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} = \frac{1}{3}$ pint

(ii) Female convicts got 'two-thirds of the male ration'.

Beef $= \frac{2}{3} \times \frac{8}{3} = \frac{16}{9} = 1\frac{7}{9}$ pounds Hardtack $= \frac{2}{3} \times \frac{14}{3} = \frac{28}{9} = 3\frac{1}{9}$ pounds

Pork $= \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$ pound Cheese $= \frac{2}{3} \times 8 = \frac{16}{3} = 5\frac{1}{3}$ ounces

Dried Peas $= \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$ pints Butter $= \frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$ ounces

Oatmeal $= \frac{2}{3} \times 2 = \frac{4}{3} = 1\frac{1}{3}$ pints Vinegar $= \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ pint

(iii) The males received two-thirds of the naval ration and females received two-thirds of males' ration.

That is, females received $\frac{2}{3}$ of $\frac{2}{3}$ or $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ of the naval ration.

To show that $\frac{4}{9}$ is slightly less than $\frac{1}{2}$ we would need to convert to a common denominator.

$$\frac{4}{9} = \frac{8}{18} \text{ while } \frac{1}{2} = \frac{9}{18}$$

So the female ration is indeed ‘slightly less than half’ the naval ration.

i.e. $\frac{8}{18} < \frac{9}{18}$

Activity 2.23

1. (a)

$$\begin{aligned}
& \frac{3}{4} \div \frac{9}{20} \\
&= \frac{3}{4} \times \frac{20}{9} \\
&= \frac{{}^{1}\cancel{3} \times \cancel{20}^{5}}{{}_{1}\cancel{4} \times \cancel{9}_{3}} \\
&= \frac{1 \times 5}{1 \times 3} \\
&= \frac{5}{3} \\
&= 1\frac{2}{3}
\end{aligned}$$

(b)

$$\begin{aligned}
& \frac{-21}{25} \div \frac{35}{30} \\
&= \frac{-21}{25} \times \frac{30}{35} \\
&= \frac{{}^{-3}\cancel{-21} \times \cancel{30}^{6}}{25 \times \cancel{35}_{\cancel{5}_{1}}} \\
&= \frac{-3 \times 6}{25} \\
&= \frac{-18}{25}
\end{aligned}$$

(c)

$$\begin{aligned}
& 4\frac{2}{5} \div 2\frac{7}{10} \\
&= \frac{22}{5} \div \frac{27}{10} \\
&= \frac{22}{5} \times \frac{10}{27} \\
&= \frac{22 \times \cancel{10}^{2}}{{}_{1}\cancel{5} \times 27} \\
&= \frac{22 \times 2}{1 \times 27} \\
&= \frac{44}{27} \\
&= 1\frac{17}{27}
\end{aligned}$$

(d)

$$\begin{aligned}
& 7\frac{5}{8} \div -3\frac{1}{2} \\
&= \frac{61}{8} \div \frac{-7}{2} \\
&= \frac{61}{8} \times \frac{-2}{7} \\
&= \frac{61 \times \cancel{-2}^{-1}}{{}_{4}\cancel{8} \times 7} \\
&= \frac{61 \times -1}{4 \times 7} \\
&= \frac{-61}{28} \\
&= -2\frac{5}{28}
\end{aligned}$$

(e)

$$\begin{aligned}
& -2\frac{1}{4} \div \frac{-3}{8} \\
&= \frac{-9}{4} \div \frac{-3}{8} \\
&= \frac{-9}{4} \times \frac{-8}{3} \\
&= \frac{{}^{-3}\cancel{-9} \times \cancel{-8}^{-2}}{{}_{1}\cancel{4} \times \cancel{3}_{1}} \\
&= \frac{-3 \times -2}{1 \times 1} \\
&= 6
\end{aligned}$$

2. Number of pieces

$$\begin{aligned}
&= 30 \div 1\frac{1}{5} \\
&= 30 \div \frac{6}{5} \\
&= \frac{30}{1} \times \frac{5}{6}
\end{aligned}$$

$$= \frac{{}^{5}\cancel{30} \times 5}{1 \times \cancel{6}_{1}}$$

$$= 25$$

Therefore 25 pieces of length $1\frac{1}{5}$ metres can be cut from the rope.

You could check your answer here by multiplying 25 by $1\frac{1}{5}$. You should of course get the original length of 30 metres.

3. Time to reach $28\frac{3}{4}$ cm will be $28\frac{3}{4} \div 1\frac{2}{3}$

$$\begin{aligned}
&= \frac{115}{4} \div \frac{5}{3} \\
&= \frac{115}{4} \times \frac{3}{5} \\
&= \frac{\overset{23}{\cancel{115}} \times 3}{4 \times \cancel{5}_{1}} \\
&= \frac{23 \times 3}{4} \\
&= \frac{69}{4} \\
&= 17\frac{1}{4}
\end{aligned}$$

$\therefore$ it would take $17\frac{1}{4}$ months to reach $28\frac{3}{4}$ cm.

↖ Did you notice this new symbol? It means therefore.

4. (a) Serving size $= \frac{{}^{1}\cancel{30}}{\underset{25}{\cancel{750}}} = \frac{1}{25}$

$\therefore$ 30 grams is $\frac{1}{25}$ of the box

(b) Protein $= \frac{1}{25} \times 90$ grams

$$\begin{aligned}
&= \frac{1 \times \overset{18}{\cancel{90}}}{{}_{5}\cancel{25} \times 1} \\
&= \frac{18}{5} \\
&= 3\frac{3}{5}
\end{aligned}$$

$\therefore$ $3\frac{3}{5}$ grams of protein in one serve.

$$\text{Fat} = \frac{1}{25} \times 21 \text{ grams}$$

$$= \frac{1 \times 21}{25 \times 1}$$

$$= \frac{21}{25}$$

$\therefore$ in one serve there are $\frac{21}{25}$ grams of fat.

$$\text{Carbohydrates} = \frac{1}{25} \times 480 \text{ grams}$$

$$= \frac{1 \times \overset{96}{\cancel{480}}}{{}_{5}\cancel{25} \times 1}$$

$$= \frac{96}{5}$$

$$= 19\frac{1}{5}$$

$\therefore$ in one serve there are $19\frac{1}{5}$ grams of carbohydrate.

$$\text{Niacin} = \frac{1}{25} \times 62 \text{ micrograms}$$

$$= \frac{1 \times 62}{25 \times 1}$$

$$= \frac{62}{25}$$

$$= 2\frac{12}{25}$$

$\therefore$ in one serving there are $2\frac{12}{25}$ micrograms of Niacin.

(c) $$\text{Number of serves} = 10 \div 2\frac{12}{25}$$

$$= \frac{10}{1} \div \frac{62}{25}$$

$$= \frac{10}{1} \times \frac{25}{62}$$

$$= \frac{{}^{5}\cancel{10} \times 25}{1 \times \underset{31}{\cancel{62}}}$$

$$= \frac{125}{31}$$

$$= 4\frac{1}{31}$$

You would need to have $4\frac{1}{31}$ serves of cereal to get the recommended daily allowance.

Activity 2.24

1. (a)

$$\begin{aligned}
& \frac{2}{3} \times \left(\frac{1}{2} + \frac{1}{3}\right) \\
= {} & \frac{2}{3} \times \left(\frac{3}{6} + \frac{2}{6}\right) \\
= {} & \frac{2}{3} \times \frac{5}{6} \\
= {} & \frac{{}^{1}\cancel{2} \times 5}{3 \times \cancel{6}_{3}} \\
= {} & \frac{5}{9}
\end{aligned}$$

(b)

$$\begin{aligned}
& 1\frac{1}{2} \div \left(\frac{3}{7} - \frac{1}{14}\right) \\
= {} & \frac{3}{2} \div \left(\frac{6}{14} - \frac{1}{14}\right) \\
= {} & \frac{3}{2} \div \frac{5}{14} \\
= {} & \frac{3}{2} \times \frac{14}{5} \\
= {} & \frac{3 \times \cancel{14}^{7}}{{}_{1}\cancel{2} \times 5} \\
= {} & \frac{21}{5} \\
= {} & 4\frac{1}{5}
\end{aligned}$$

(c)

$$\begin{aligned}
& \frac{3}{5} - 2\frac{2}{5} \times \frac{19}{4} \\
= {} & \frac{3}{5} - \frac{12}{5} \times \frac{19}{4} \\
= {} & \frac{3}{5} - \frac{{}^{3}\cancel{12} \times 19}{5 \times \cancel{4}_{1}} \\
= {} & \frac{3}{5} - \frac{57}{5} \\
= {} & \frac{-54}{5} \\
= {} & -10\frac{4}{5}
\end{aligned}$$

(d)

$$\begin{aligned}
& \frac{3}{4} \div \frac{6}{5} + \frac{6}{8} \times 3\frac{2}{3} \\
= {} & \frac{3}{4} \div \frac{6}{5} + \frac{6}{8} \times \frac{11}{3} \\
= {} & \frac{3}{4} \times \frac{5}{6} + \frac{{}^{2}\cancel{6} \times 11}{8 \times \cancel{3}_{1}} \\
= {} & \frac{{}^{1}\cancel{3} \times 5}{4 \times \cancel{6}_{2}} + \frac{{}^{1}\cancel{2} \times 11}{{}_{4}\cancel{8} \times 1} \\
= {} & \frac{5}{8} + \frac{11}{4} \\
= {} & \frac{5}{8} + \frac{22}{8} \\
= {} & \frac{27}{8} \\
= {} & 3\frac{3}{8}
\end{aligned}$$

(e)

$$\begin{aligned}
& \left(\frac{3}{5}\right)^{2} - \left(\frac{4}{5} + \frac{1}{2}\right) \\
= {} & \frac{3}{5} \times \frac{3}{5} - \left(\frac{8}{10} + \frac{5}{10}\right) \\
= {} & \frac{9}{25} - \frac{13}{10} \\
= {} & \frac{18}{50} - \frac{65}{50} \\
= {} & \frac{-47}{50}
\end{aligned}$$

2.

$$\begin{aligned}
\text{Non-smokers} &= 1 - \left(\frac{2}{5} + \frac{1}{3}\right) \\
&= 1 - \left(\frac{6}{15} + \frac{5}{15}\right) \\
&= 1 - \frac{11}{15} \\
&= \frac{15}{15} - \frac{11}{15} \\
&= \frac{4}{15}
\end{aligned}$$

Now $\frac{4}{15} \times 750$

$$= \frac{4 \times \overset{50}{\cancel{750}}}{{}_{1}\cancel{15} \times 1}$$

$= 200$

$\therefore$ 200 of the teenagers were non-smokers.

3. Change $= 100 - \left(3\frac{1}{2} \times 4 + 4 \times 5 + 12 + 1\frac{3}{4} \times 8\right)$

$$= 100 - \left(\frac{7}{{}_{1}\cancel{2}} \times \cancel{4}^{2} + 20 + 12 + \frac{7}{\cancel{4}_{1}} \times \cancel{8}^{2}\right)$$

$= 100 - (14 + 20 + 12 + 14)$

$= 100 - 60$

$= 40$

The Browns would receive \$40 change.

Activity 2.25

1.

	millions	**hundred thousands**	**ten thousands**	**thousands**	**hundreds**	**tens**	**units**	**•**	**tenths**	**hundredths**	**thousandths**	**ten thousandths**
(a)					5	2	4	•	1	9		
(b)				2	0	0	0	•	0	0	4	
(c)	–	7	9	5	0	0	0					
(d)		–	6	6	0	5	5	•	3	2		
(e)							0	•	3	0	6	

2. (a) $79\underline{5}.2\underline{4}$ The 5 represents 5 units. The 4 represents 4 hundredths.

   (b) $\underline{2}004.\underline{6}$ The 2 represents 2 thousands. The 6 represents 6 tenths.

   (c) $\underline{8}2.30\underline{5}$ The 8 represents 8 tens. The 5 represents 5 thousandths.

(d) $-\underline{6}\,953\,000.0\underline{1}$ The 6 represents 6 millions. The 1 represents 1 hundredth.

(e) $-\underline{1}2\,567.005\,\underline{7}$ The 1 represents 1 ten thousand. The 7 represents 7 ten thousandths.

Activity 2.26

1. (a) 576.205 ≈ 576.21 to the nearest hundredth.

(b) 75 201.3 ≈ 75 201 to the nearest unit.

(c) –0.008 ≈ –0.01 to the nearest hundredth.

(d) 67.345 67 ≈ 67.3 to one decimal place.

(e) –6 399.998 ≈ –6 400.00 to two decimal places.

Note here that although these decimal places end up as zeros they must be included to show that we have rounded to two decimal places.

2. (a) 2 576.205 ≈ 3 000

(b) 201.3 ≈ 200

(c) –0.028 ≈ –0.03

(d) 97.345 67 ≈ 100

(e) –6 009 ≈ –6 000

Activity 2.27

1. (a) 9.56 + 2.589 + 13.81

Estimate	Calculate	Check
9.56 ≈ 10	9.56	
2.589 ≈ 3	2.589	Yes, looks reasonable.
13.81 ≈ 10	+ 13.81	
	1 1 1	
10 + 3 + 10 = 23	25.959	

(b) 15.67 – 2.5

Estimate	Calculate	Check
15.67 ≈ 20	15.67	
2.5 ≈ 3	– 2.5	Yes, looks reasonable.
20 – 3 = 17	13.17	

(c) 148 + 0.003 + 2.607 9

Estimate		Calculate	Check
148	≈ 100	148	
0.003	≈ 0.003	0.003	Yes, looks reasonable.
2.607 9	≈ 3	+ 2.607 9	
		1 1	
100 + 0.003 + 3	= 103.003	150.610 9	

(d) 3.68 – 89.2 Note that the second number is larger so the answer will be negative

Estimate		Calculate	
		3.68 – 89.2 = –(89.2 – 3.68)	Take smaller number from larger.
3.68	≈ 4	8 11 $\cancel{1}$ 10	
89.2	≈ 90	$8\cancel{9}.\cancel{2}\cancel{0}$	
		– 3.68	Check
4 – 90	= –86	85.52	Looks reasonable.
		∴ 3.68 – 89.2 = –85.52	

(e) –8 974.2 + 356.47 The answer will be negative.

Estimate		Calculate	
		–8 974.2 + 356.47 = –(8 974.2 – 356.47) Take smaller number from larger	
		13 11 6 $\cancel{3}$ $\cancel{1}$ 10	
–8 974.2	≈ –9 000	$8\ 9\cancel{7}\cancel{4}.\cancel{2}\cancel{0}$	Check
356.47	≈ 400	– 356.47	Looks reasonable.
–9 000 + 400	= –8 600	8 617.73	
		∴ –8 974.2 + 356.47 = –8 617.73	

2. Teenagers raised \$18.57 + \$24.32 + \$9.84

Estimate		Calculate	Check
18.57	≈ 20	18.57	
24.32	≈ 20	24.32	
9.84	≈ 10	+ 9.84	Yes, looks reasonable.
		2 1 1	
20 + 20 + 10	= 50	52.73	

3. Poison remaining = 2 – 0.15 litres

$$\begin{array}{r} \overset{1}{\cancel{2}}.\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{10}{\cancel{0}} \\ -\,0.15 \\ \hline 1.85 \end{array}$$

∴ 1.85 litres of poison remain

4. (a)

	Weight (kilograms)	**Weight loss**
Start	88.904	–
10th day	87.772	1.132
20th day	86.592	1.18
30th day	84.036	2.556

(b) Total weight loss = 1.132 + 1.18 + 2.556

Estimate	Calculate	Check
1.132 ≈ 1	1.132	
1.18 ≈ 1	1.18	Yes, looks reasonable.
2.556 ≈ 3	+ 2.556	
	1	
1 + 1 + 3 = 5	4.868	

The patient lost 4.868 kilograms

(c) To the nearest tenth of a kilogram the patient lost 4.9 kilograms.

5. Income from disco = \$64 + \$17.40 + \$16.77

$$\begin{array}{r} 64 \\ 17.40 \\ +\,16.77 \\ {\scriptstyle 1\,1} \\ \hline 98.17 \end{array}$$

Income was \$98.17

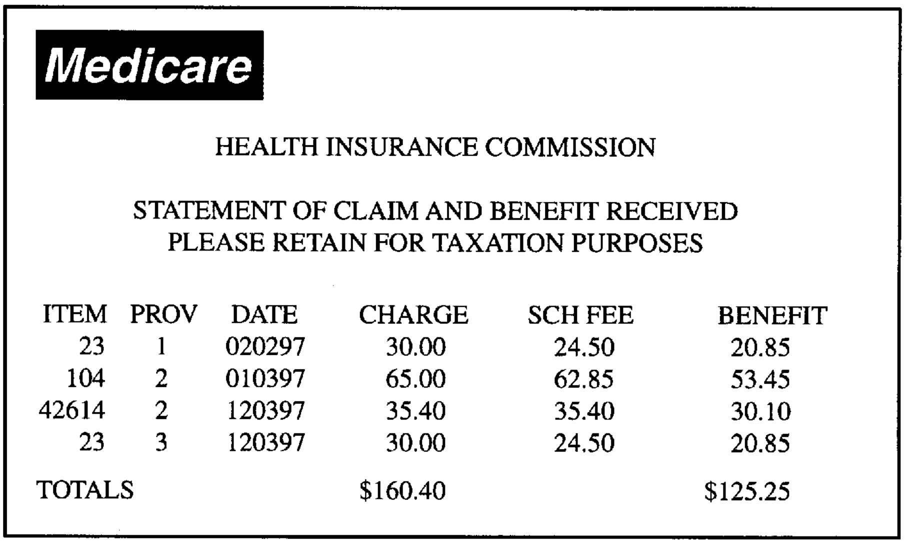
Youth Club had spent $100

So Youth Club had to pay $100 – $98.17

$$\begin{array}{r} \cancel{1}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}.\overset{9}{\cancel{0}}\overset{10}{\cancel{0}} \\ -\ 98.17 \\ \hline 1.83 \end{array}$$

The Youth Club has to pay $1.83

6. (a)



Medicare

HEALTH INSURANCE COMMISSION

STATEMENT OF CLAIM AND BENEFIT RECEIVED
PLEASE RETAIN FOR TAXATION PURPOSES

ITEM	PROV	DATE	CHARGE	SCH FEE	BENEFIT
23	1	020297	30.00	24.50	20.85
104	2	010397	65.00	62.85	53.45
42614	2	120397	35.40	35.40	30.10
23	3	120397	30.00	24.50	20.85
TOTALS			$160.40		$125.25

(b) Patient paid $160.40 – $125.25

$$\begin{array}{r} 1\overset{5}{\cancel{6}}\overset{10}{\cancel{0}}.\overset{3}{\cancel{4}}\overset{10}{\cancel{0}} \\ -\ \underline{125.25} \\ 35.15 \end{array}$$

The patient had to pay $35.15

(c) Item 23: $30 – $24.50 = $5.50

Item 104: $65 – $62.85 = $2.15

Item 42614: $35.40 – $35.40 = $0

Activity 2.28

1. (a) 52.6×3.95

Estimate	Calculate	Check
$52.6 \approx 50$	52.6	
$3.95 \approx 4$	$\underline{\times\ 3.95}$	Yes, looks reasonable.
	1 3	
$50 \times 4 = 200$	2500	
	1 5	
	45840	
	1	
	156800	
	1 1 2	
	$\overline{207.770}$	Three decimal places in question so put three in answer.
$\therefore\ 52.6 \times 3.95 = 207.77$		There is no need to write the last zero.

(b) -2.05×20 A negative multiplied by a positive equals a negative.

Estimate	Calculate	
	$-2.05 \times 20 = -(2.05 \times 20)$	
$-2.05 \approx -2$	2.05	
$20 = 20$	$\underline{\times\ 20}$	
	1	
$-2 \times 20 = -40$	$\underline{4\,000}$	
	41.00	Two decimal places in question so two in answer.

$\therefore$ $-2.05 \times 20 = -41$

(c) $0.56 \times {}^{-}1.23$ Positive multiplied by a negative equals a negative.

Estimate	Calculate	
	$0.56 \times {}^{-}1.23 = -(0.56 \times 1.23)$	
$0.56 \approx 0.6$	1.23	
$-1.23 \approx -1$	$\underline{\times\ 0.56}$	
	1 1	
$0.6 \times {}^{-}1 = -0.6$	628	
	1 1	
	$\underline{5050}$	
	0.6888	Four decimal places in question so four in the answer.

$\therefore$ $0.56 \times {}^{-}1.23 = -0.6888$

(d) 145 × 1.58

Estimate	Calculate	Check
145 ≈ 100	1.58	
1.58 ≈ 2	× 145	Yes, looks reasonable.
	2 4	
100 × 2 = 200	550	
	2 3	
	4020	
	15800	
	1 1 1	
	229.10	

∴ 145 × 1.58 = 229.1 The final zero is not necessary.

(e) –0.025 × ¯3.6 Answer will be positive

Estimate	Calculate	Check
–0.025 ≈ –0.03	3.6	
–3.6 ≈ –4	× 0.025	Yes, looks reasonable.
	3	
–0.03 × ¯4 = 0.12	150	
	1	
	620	
	1	
	0.0900	We must insert zeros to get the four decimal places required.

∴ –0.025 × ¯3.6 = 0.09

2. 2 years = 24 months. Olga pays 24 × $37.45

(a) Estimate	(b) Calculate	Check
24 ≈ 20	37.45	
37.45 ≈ 40	× 24	Yes, looks reasonable.
	2 1 2	
20 × 40 = 800	12860	
	1 1	
	64800	
	1	
	898.80	

Olga pays $898.80 for the stereo.

3. Over 20 days the amount of drug used will be: 1.5 × 12 × 20 millilitres

Estimate	Calculate		Check
1.5 ≈ 2	1.5	18	
12 ≈ 10	× 12	× 20	Yes, looks reasonable.
	1	1	
20 = 20	20	260	
2 × 10 × 20 = 400	150	360	
	18.0		

∴ 360 millilitres of the drug is used in 20 days.

4. Total thickness = $367 \times 0.1 + 2 \times 0.2$

Estimate	Calculate	Check
$367 \approx 400$	367	
$0.1 = 0.1$	$\underline{\times\, 0.1}$	
$400 \times 0.1 + 2 \times 0.2$	36.7	Yes, looks reasonable.
$= 40 + 0.4 = 40.4$	$367 \times 0.1 + 2 \times 0.2$	
	$= 36.7 + 0.4$	
	$= 37.1$	

The book will be 37.1 millimetres thick.

5. (a) For a 40 hour week salary = $40 \times \$14.68$

$$\text{Estimate:}\quad \begin{array}{rcl} 40 & = & 40 \\ 14.68 & \approx & 10 \\ 40 \times 10 & = & 400 \end{array}$$

Irving earns about \$400

(b) Calculate: 40×14.68

$$\begin{array}{r} 14.68 \\ \underline{\times\, 40} \\ {\scriptstyle 1\,2\,3} \\ \underline{46420} \\ 587.20 \end{array}$$

Irving earns \$587.20 for a 40 hour week.

(c) For a 50 hour week Irving earns $40 \times \$14.68 + 10 \times 1\frac{1}{2} \times \$14.68$

Estimate: $40 \times 10 + 10 \times 2 \times 10$

$= 400 + 200$

$= 600$

Calculate: $40 \times 14.68 + 10 \times 1\frac{1}{2} \times 14.68$

$= 587.20 + 15 \times 14.68$

$= 587.20 + 220.20$

$= 807.40$

For a 50 hour week Irving earns \$807.40

6. (a) (i) Individually 20 × \$5 = \$100

   (ii) To purchase 20, I need two packs of 10
   2 × \$47.50 = \$95

   (iii) Saving = \$100 – \$95 = \$5

   (b) Eight 3 kg satchels = 8 × \$7.70 = \$61.60

   100 C5 envelopes = 10 × \$25.50 = \$255.00

   100 DL envelopes = 2 × \$127.50 = \$255.00

   Total order = \$571.60

7. Total water used = 3 981 – 3 619 kilolitres

   = 362 kilolitres

   This is above the 324 kilolitres so some will be charged at the second tier.

   2nd Tier 362 – 324 = 38 kilolitres

   Water bill will be 324 × \$0.35 + 38 × \$1.00

   = \$113.40 + \$38.00

   = \$151.40

   The water charge at the Kennedy residence is \$151.40

8. (a) \$4 356 No tax payable

   (b) \$34 780 tax is \$3 060 plus 34¢ for each \$1 over \$20 700

   Amount in excess of \$20 700 = \$34 780 – \$20 700 = \$14 080

   34¢ in the \$1 = \$14 080 × \$0.34 (both numbers must be in dollars)

   = \$4 787.20

   Total tax = \$3 060 + \$4 787.20

   = \$7 847.20

(c) \$48 940 tax is \$8 942 + 43¢ for each \$1 over \$38 000

Amount in excess of \$38 000 = \$48 940 – \$38 000 = \$10 940

43¢ in the \$1 = \$10 940 × \$0.43

= \$4 704.20

Total tax = \$8 942 + \$4 704.20

= \$13 646.20

(d) \$56 200 tax is \$14 102 + 47¢ for each \$1 over \$50 000

Amount in excess of \$50 000 = \$56 200 – \$50 000 = \$6 200

47¢ in the \$1 = \$6 200 × \$0.47

= \$2 914

Total tax = \$14 102 + \$ 2 914

= \$17 016

Activity 2.29

1. (a) $0.672 \div 3.2$

Estimate	Calculate	Check
$0.672 \approx 0.7$	$0.672 \div 3.2 = 6.72 \div 32$	
$3.2 \approx 3$	$\begin{array}{r} 0.21 \\ 32\overline{)6.72} \\ \underline{6\,4} \\ 32 \\ \underline{32} \\ 0 \end{array}$	Yes, looks reasonable.
$0.7 \div 3$ is between 0.2 and 0.3		

$\therefore\ 0.672 \div 3.2 = 0.21$

(b) $-8.326 \div 0.23$ Answer will be negative

Estimate	Calculate	Check
$-8.326 \approx -8$	$-8.326 \div 0.23 = -832.6 \div 23$	
	$= -(832.6 \div 23)$	

$0.23 \approx 0.2$

$-8 \div 0.2 = -80 \div 2 = -40$

$$\begin{array}{r} 36.2 \\ 23\overline{)832.6} \\ \underline{69} \\ 142 \\ \underline{138} \\ 4\ 6 \\ \underline{4\ 6} \\ 0 \end{array}$$

Yes, looks reasonable.

$\therefore -8.326 \div 0.23 = -36.2$

(c) $0.002\ 88 \div 0.012$

Estimate Calculate Check

$0.002\ 88 \approx 0.003$

$0.012 \approx 0.01$

$0.003 \div 0.01 = 0.3 \div 1$

$= 0.3$

$0.002\ 88 \div 0.012 = 2.88 \div 12$

$$\begin{array}{r} 0.24 \\ 12\overline{)2.88} \\ \underline{2\ 4} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

Yes, looks reasonable.

$\therefore 0.002\ 88 \div 0.012 = 0.24$

(d) $-16.605 \div {}^{-}36.9$ Answer will be positive

Estimate Calculate Check

$-16.605 \approx -20$

$-36.9 \approx -40$

$-2\cancel{0} \div {}^{-}4\cancel{0} = 0.5$

$-16.605 \div {}^{-}36.9 = -166.05 \div {}^{-}369$

$$\begin{array}{r} 0.45 \\ 369\overline{)166.05} \\ \underline{147\ 6} \\ 18\ 45 \\ \underline{18\ 45} \\ 0 \end{array}$$

Yes, looks reasonable.

$\therefore -16.605 \div -36.9 = 0.45$

(e) $810 \div 40$

Estimate

$$\begin{array}{rcl} 810 & \approx & 800 \\ 40 & = & 40 \\ 80\cancel{0} \div 4\cancel{0} & = & 20 \end{array}$$

Calculate

$$810 \div 40$$

$$= 81 \div 4$$

Cancel zeros.

$$\begin{array}{r} 20.25 \\ 4\overline{)81.00} \\ \underline{8} \\ 01 \\ \underline{0} \\ 1\,0 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

We must enter the decimal point and some zeros.

Check

Yes, looks reasonable.

$\therefore\ 810 \div 40 = 20.25$

2. Number of pieces = $6 \div 0.75 = 600 \div 75$

Estimate

$$\begin{array}{rcl} 600 & = & 600 \\ 75 & \approx & 80 \end{array}$$

$60\cancel{0} \div 8\cancel{0}$ is between 7 and 8

Calculate

$$\begin{array}{r} 8 \\ 75\overline{)600} \\ \underline{600} \\ 0 \end{array}$$

$\therefore$ the pipe can be cut into 8 pieces.

3. Time to pay off balance = $\$565.92 \div \$23.58 = 56\,592 \div 2\,358$

Estimate

$$\begin{array}{rcl} 56\,592 & \approx & 60\,000 \\ 2\,358 & \approx & 2\,000 \\ 60\,\cancel{000} \div 2\,\cancel{000} & = & 30 \end{array}$$

Calculate

$$\begin{array}{r} 24 \\ 2\,358\overline{)56\,592} \\ \underline{47\,16} \\ 9\,432 \\ \underline{9\,432} \\ 0 \end{array}$$

Check

Yes, looks reasonable.

$\therefore$ it would take 24 months or 2 years to pay off the account.

4. Number of doses = 12.5 ÷ 1.25 = 1 250 ÷ 125

Estimate	Calculate	Check
$1\,250 \approx 1\,000$	10	
$125 \approx 100$	$125\overline{)1\,250}$	Yes, looks reasonable
$1\,0\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} = 10$	1 25	
	00	

∴ the vial contains 10 single doses.

5. 280 centimetres = 280 ÷ 2.54 inches = 28 000 ÷ 254

Estimate	Calculate
$28\,000 \approx 30\,000$	1 1 0.2 3 6 …
$254 \approx 300$	$254\overline{)2\,8\,0\,0\,0.0\,0\,0}$
$30\,0\cancel{0}\cancel{0} \div 3\cancel{0}\cancel{0} = 100$	2 5 4
	2 6 0
	2 5 4
	6 0 0
	5 0 8
	9 2 0
	7 6 2
	1 5 8 0
	1 5 2 4
	5 6

This would keep on going. We should stop and round off.

∴ 280 centimetres is approximately equal to 110.24 inches

6. Profit = 154 × ($5.00 – $0.50) = 154 × $4.50

Estimate	Calculate	Check
$154 \approx 200$	154	
$4.50 \approx 5$	4.50	Yes, looks reasonable.
	$^{2\,2}$	
$200 \times 5 = 1\,000$	5500	
	$^{2\,1}$	
	40600	
	^{1}	
	693.00	

The clinic's profit is $693.00

Activity 2.30

1. (a) $\frac{1}{8} = 1 \div 8 = 0.125$

(b) $-\frac{1}{4} = -1 \div 4 = -0.25$

(c) $-\frac{7}{10} = -7 \div 10 = -0.7$

(d) $\frac{1}{3} = 1 \div 3 = 0.333\ldots = 0.\dot{3}$

(e) $\frac{13}{7} = 13 \div 7 = 1.857142857\ldots = 1.\dot{8}5714\dot{2}$

(f) $\frac{-25}{11} = -25 \div 11 = -2.272727\ldots = -2.\dot{2}\dot{7}$

(g) $1\frac{3}{4} = \frac{7}{4} = 7 \div 4 = 1.75$

2. (a) $0.7 = \frac{7}{10}$

(b) $0.8 = \frac{\cancel{8}^{4}}{\cancel{10}_{5}} = \frac{4}{5}$

(c) $0.35 = \frac{\overset{7}{\cancel{35}}}{\underset{20}{\cancel{100}}} = \frac{7}{20}$

(d) $-0.58 = \frac{\overset{-29}{\cancel{-58}}}{\underset{50}{\cancel{100}}} = \frac{-29}{50}$

(e) $0.075 = \frac{\overset{3}{\cancel{75}}}{\underset{40}{\cancel{1\,000}}} = \frac{3}{40}$

(f) $-0.625 = \frac{\overset{-25}{\cancel{-625}}}{\underset{40}{\cancel{1\,000}}} = \frac{\overset{-5}{\cancel{-25}}}{\cancel{40}_{8}} = \frac{-5}{8}$

(g) $4.02 = \frac{\overset{201}{\cancel{402}}}{\underset{50}{\cancel{100}}} = \frac{201}{50} = 4\frac{1}{50}$

Solutions to a taste of things to come

1. (a) Credit limit is $4 000.

This means that you cannot owe more than $4 000 on this account.

(b) The closing balance or minimum payment should have been made on or before 28 February 1997.

(c)

Opening balance	Total credits (CR)	Total debits (DR)	Credit and other charges	Closing balance A$
$1 246.73	$178	$311.66	$20.34	$1 400.73

2. (a) Corpses showing toxic levels of drugs = $\frac{70}{374} = \frac{35}{187}$

(b) (i) toxic levels of analgesics = $\frac{13}{70}$

(ii) toxic levels of barbiturates = $\frac{9}{70}$

(iii) toxic levels of pentobarbitone = $\frac{5}{70} = \frac{1}{14}$

(c) More than one drug present = $\frac{46}{374} = \frac{23}{187}$

(d) If 70 showed drugs then 374 – 70 had no drugs. That is 304 showed no drugs.

Fraction with no drugs = $\frac{304}{374} = \frac{152}{187}$

3. (a) You will have a variety of responses for this question.

(b) The correct interpretation is that the chance of developing breast cancer **increases** with age.

Lower-risk group shouldn't let lump go unnoticed		
Age	**Group size for 1 to develop breast cancer per year**	**Chance of getting breast cancer as a fraction**
40	1 041	$\frac{1}{1\ 041}$
45	653	$\frac{1}{653}$
50	610	$\frac{1}{610}$
55	515	$\frac{1}{515}$
60	440	$\frac{1}{440}$
65	392	$\frac{1}{392}$
70	382	$\frac{1}{382}$

So at 40 a woman has 1 chance in 1 041 of developing breast cancer, while at 70 she has 1 chance in 382. The first fraction is the smallest fraction and gives the least chance of developing breast cancer, while the last fraction $\frac{1}{382}$ is the largest fraction and consequently the greatest risk.

How did the people you surveyed go?

(c) Following is a sample of what you might have written.

Information collected from a major breast screening unit has clearly identified the risks of a woman developing breast cancer. The older you are the greater the risk of developing breast cancer. A woman of 40 has 1 chance in 1 041 of developing breast cancer, while at 70 she has 1 chance in 382. You could think of this in another way. If you collected together 1 041 forty year old women, you would expect one of these to have breast cancer, but you would only need to gather 382 seventy year old women together to find one person that has breast cancer.

Solutions to post-test

1. (a) $765 \div 15 + 822$

Estimate	Calculate	Check
$\approx 800 \div 20 + 800$	$765 \div 15 + 822$	
$= 40 + 800$	$= 51 + 822$	Yes, looks reasonable.
$= 840$	$= 873$	

(b) $89 + 21 - 48 \times 23$

Estimate	Calculate	Check
$\approx 90 + 20 - 50 \times 20$	$89 + 21 - 48 \times 23$	
$= 90 + 20 - 1\,000$	$= 89 + 21 - 1\,104$	Yes, looks reasonable.
$= 110 - 1\,000$	$= 110 - 1\,104$	
$\approx 100 - 1\,000$	$= -994$	
$= -900$		

(c) $591 \times 376 + 523$

Estimate	Calculate	Check
$\approx 600 \times 400 + 500$	$591 \times 376 + 523$	
$= 240\,000 + 500$	$= 222\,216 + 523$	Yes, looks reasonable.
$= 240\,500$	$= 222\,739$	

(d) $895(622 + 479)$

Estimate	Calculate	Check
$\approx 900(600 + 500)$	$895(622 + 479)$	
$= 900(1\,100)$	$= 895 \times 1\,101$	Yes, looks reasonable.
$\approx 900 \times 1\,000$	$= 985\,395$	
$= 900\,000$		

(e) $4\,763 + 395 \div 5 \times 16$

Estimate	Calculate	Check
$\approx 5\,000 + 400 \div 5 \times 20$	$4\,763 + 395 \div 5 \times 16$	
$= 5\,000 + 80 \times 20$	$= 4\,763 + 79 \times 16$	Yes, looks reasonable.
$= 5\,000 + 1\,600$	$= 4\,763 + 1\,264$	
$= 6\,600$	$= 6\,027$	

2. (a) $(8.47)^2 \times 6.23$

Estimate	Calculate	Check
$\approx 8^2 \times 6$	$8.47\ x^2 \times 6.23 =$	
$= 64 \times 6$		
$\approx 60 \times 6$	≈ 446.95	Looks reasonable.
$= 360$		

(b) $\sqrt{\frac{584}{73.8}}$

Estimate	Calculate	Check
$\approx \sqrt{\frac{60\cancel{0}}{7\cancel{0}}}$	$584 \div 73.8 = \sqrt{\ }$	
$\approx \sqrt{9}$	≈ 2.81	Looks reasonable.
$= 3$		

(c) $(\sqrt{8.25})^2 - 85.72$

Estimate	Calculate	Check
$\approx (\sqrt{8})^2 - 90$	$8.25\ \sqrt{\ }\ x^2 - 85.72 =$	
$= 8 - 90$	$= -77.47$	Looks reasonable.
$= -82$		

(d) $\frac{6.4 \times 34}{12.4}$

Estimate	Calculate	Check
$\approx \frac{6 \times 30}{10}$	$6.4 \times 34 \div 12.4 =$	
$= \frac{180}{10}$	≈ 17.55	Looks reasonable.
$= 18$		

(e) $(48 \div 8.4)^3$

Estimate	Calculate	Check
$= (50 \div 8)^3$	$(48 \div 8.4)\ x^y\ 3 =$	
$\approx 6^3$		
$= 6 \times 6 \times 6$	≈ 186.59	Looks reasonable.
$= 36 \times 6$		
$\approx 40 \times 6$		
$= 240$		

3.

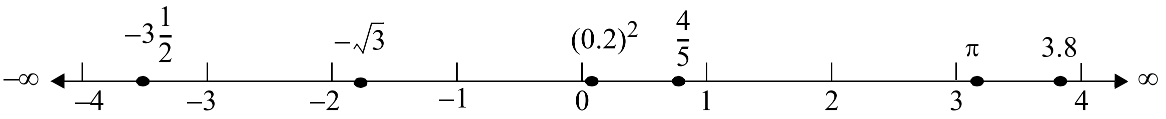
Fraction	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.125	0.2	0.25	$0.\dot{3}$	0.5	$0.\dot{6}$	0.75

4. (a)

$(0.2)^2 = 0.04$ $\qquad -\sqrt{3} \approx {}^{-}1.73$

$-3\frac{1}{2} = -3.5$ $\qquad 3.8 = 3.8$

$\pi \approx 3.14$ $\qquad \frac{4}{5} = 0.8$



(b) You should press the following keys to find the sum

0 • 2 x^2 + 3 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 $^{+}/_{-}$ + π

+ 3 $\sqrt{}$ $^{+}/_{-}$ + 3 • 8 + 4 $a\frac{b}{c}$ 5 =

The display should read 2.549541846…

Rounded to the nearest thousandth = 2.550

5. (a) 94.7 – 2.6 – 1.2 + 0.4 – 0.9 + 1.5 – 2.7 – 0.1 + 0.4 = 89.5

The person weighs 89.5 at the end of eight weeks.

(b) Total weight lost = 94.7 – 89.5 = 5.2

Total weight lost was 5.2 kilograms.

(c) Needs to lose 89.5 – 68 = 21.5

This person needs to lose a further 21.5 kilograms.

6. (a) General rate = $\frac{1}{2} \times \$0.017\,29 \times \$42\,500$ = \$367.41

Park and Bushland charge = $\frac{1}{2} \times \$15.00$ = \$7.50

Water access charge = $\frac{1}{2} \times \$235.00$ = \$117.50

Sewerage charge = $\frac{1}{2} \times \$134.00$ = \$67.00

Cleaning charge = $\frac{1}{2} \times \$70.00$ = \$35.00

Fire levy = $\frac{1}{2} \times \$95.20$ = \$47.60

Total Rates for $\frac{1}{2}$ year = \$642.01

(b) General rate = $\frac{1}{2} \times \$0.020\,29 \times \$82\,540$ = \$837.37

Park and Bushland charge = $\frac{1}{2} \times \$15.00$ = \$7.50

Water access charge = $\frac{1}{2} \times \$235.00$ = \$117.50

Sewerage charge = $\frac{1}{2} \times \$134$ 1st pedestal + $\frac{1}{2} \times 3 \times \$124$ = \$253.00

Cleaning charge = $\frac{1}{2} \times \$170$ = \$85.00

Fire levy = $\frac{1}{2} \times \$95.20$ = \$47.60

Total Rates for $\frac{1}{2}$ year = \$1 347.97

7. (a) Most people moved to Queensland.

(b)

Arrived in	**Left NSW for**	**Left Vic for**	**Left Qld for**	**Left SA for**	**Left WA for**	**Left Tas for**	**Left NT for**	**Left ACT for**	**TOTAL**
Qld	53 110	31 069		8 900	7 548	4 147	6 205	3 894	114 873
NSW		24 182	34 184	6 445	6 770	2 442	2 532	10 496	87 051
Vic	19 442		14 628	6 776	5 191	3 125	2 052	2 231	53 445
WA	7 669	7 472	6 618	3 759		1 626	3 587	1 006	31 737
SA	5 682	7 446	4 772		2 542	755	2 783	724	24 704
ACT	10 425	2 603	2 876	1 132	985	319	739		19 079
NT	3 430	2 922	4 805	3 202	2 606	443		594	18 002
Tas	2 272	2 699	2 216	1 019	1 173		452	311	10 142
TOTAL	102 030	78 393	70 099	31 233	26 815	12 857	18 350	19 256	359 033

NSW had the greatest number of people leave.

(c)

State	Arrived	Departed	Nett Migration
Qld	114 873	70 099	44 774
NSW	87 051	102 030	–14 979
Vic	53 445	78 393	–24 948
WA	31 737	26 815	4 922
SA	24 704	31 233	–6 529
ACT	19 079	19 256	–177
NT	18 002	18 350	–348
Tas	10 142	12 857	–2 715

All states except Queensland and WA had negative nett migration.

(d) The Australian Capital Territory (ACT) had the least change in population with a drop of only 177.

(e)

Left NSW for SA = 5 682

Total leaving NSW = 102 030

Fraction leaving NSW for SA $= \frac{5\ 682}{102\ 030} = \frac{947}{17\ 005}$

(f) Fraction arriving in WA from Queensland $= \frac{6\ 618}{31\ 737} = \frac{2\ 206}{10\ 579}$